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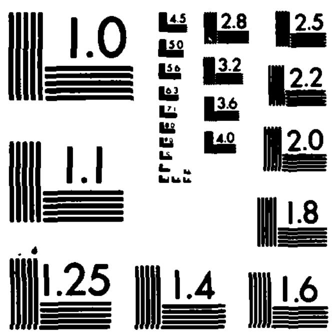

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER AFOSR-TR- 84-0254	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Investigations of Vacuum Ultraviolet and Soft X-ray Lasers		5. TYPE OF REPORT & PERIOD COVERED Final Report 5/15/82 - 5/14/83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ahmet Elci		8. CONTRACT OR GRANT NUMBER(s) AFOSR-81-0128
9. PERFORMING ORGANIZATION NAME AND ADDRESS Institute for Modern Optics, Dept. of Physics and Astronomy, Univ. of NM, Albuquerque, NM 87131		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2301/A8
11. CONTROLLING OFFICE NAME AND ADDRESS Airforce Office of Scientific Reseach (AFOSR)/NP Bolling AFB, DC 20332		12. REPORT DATE March, 1984
		13. NUMBER OF PAGES 68
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

A

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

X-ray Lasers, Free Electron Laser, Nuclear Spectroscopy

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The regimes in which an X-ray free electron laser can be operated are discussed. One possible use of an X-ray laser to nuclear spectroscopy is described. Intense field effects on nuclear beta decay are analyzed.

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AFOSR-TR- 84-0254

Investigations of Vacuum Ultraviolet and Soft X-Ray Lasers

Report for
Grant Period 1982-1983

Prepared by

A. Elci

Principle Investigator

Marlan O. Scully

Institute for Modern Optics
Department of Physics and Astronomy
University of New Mexico
Albuquerque, New Mexico 87131

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TABLE OF CONTENTS

I. INTRODUCTION

Prior to the grant period 5/15/82 - 5/14/83, the efforts of our group concentrated on charge-exchange schemes to obtain lasing in the X-ray region. During the period 5/15/82 - 5/14/83, the efforts of the group were directed towards alternative schemes, such as the free-electron laser, to obtain X-ray lasing. Possible uses of X-ray lasers in novel areas, such as in nuclear phenomena, were also investigated, which led to rigorous analyses of some earlier work concerning nuclear beta decay, as well as careful considerations of intense field behavior of quantum systems.

The following report consists of two parts. The first part summarizes the initial work concerning the possibility of scaling the free-electron laser to X-ray wavelengths. The second part summarizes the work dealing with the possible uses of X-ray lasers in nuclear phenomena, and with correct descriptions of quantum systems in intense fields. The papers which detail these results either in published or in preprint form, are reproduced as appendices.

II. SCALING OF FREE-ELECTRON LASER TO X-RAY WAVELENGTHS

A difficulty in extending the operation of lasers based on atomic transitions down to X-ray wavelengths is the short lifetime of excited states, due to radiative and Auger transitions. This difficulty is not present in the case of free-electron lasers (FEL), since the energy source for the lasing is stored stably (for hours, in the case of storage rings) in the form of electron kinetic energy. Moreover, the FEL is intrinsically a swept-gain device, normally operating with overlapping picosecond pulses of electrons and laser radiation. Thus, a gain length of many meters is possible in a mostly empty cavity, since the gain pulse (electrons) moves at nearly the same speed as the laser pulse. The FEL wavelength is continuously tunable by changing the electron energy, and so can be matched to available Bragg crystal resonators.

Two possible ways to achieve short-wavelength operation of the FEL have been considered. One is to use high-energy (GeV) electrons in conjunction with a long conventional wiggler (or a GW microwave field in a waveguide). The second is to use low-energy (10 MeV) electrons passing through a counter-propagating high-power infrared laser pulse. The first regime can give very large gain (operation without a resonator is a possibility), but requires a wiggler tens to hundreds of meters ion length, unless a good quality Bragg resonator can be developed. Electron beam quality is the most important limitation. Storage-ring operation may be feasible down to about 60 Å. Shorter wavelengths (down to a few Å) are possible in principle using a linear accelerator, but suitable accelerators have not been developed.

The second regime using a high-power laser pulse can give moderate (40%) gain. In this regime the quantum recoil can be as large or larger than the homogeneous bandwidth, so that quantum effects are definitely important. The quantum effects, though physically interesting, are in general adverse, reducing the gain and limiting the saturation power to about one photon per electron. The correct gain can be calculated using semiclassical theory (quantizing the electrons, but not the field). Another important feature of this type of FEL is the slowing of the electrons as they pass through the high-intensity beam waist of the counterpropagating Gaussian beam. This slowing is due to transfer of electron kinetic energy into transverse motion (mass-shift effect) and spreads the gain over a bandwidth dependent on the pump laser power (but not on the size of the beam waist). In general, electrons in an appropriate energy range are resonant at one point entering and at one point leaving the focal region, and interference between radiation emitted at these two points gives a rapidly oscillating homogeneous spectral lineshape. The highest peak of the lineshape curve is at the low-frequency end of the curve, and corresponds to electrons which are resonant at the beam waist. The frequency of this peak depends on the

power of the pump laser. A difficulty with this regime is to devise a cavity for injecting and recirculating the terawatt pump pulse and to keep the electrons resonant for many passes if the pump pulse is decaying.

The work summarized above is described in the paper by J. Gea-Banachloche, G. T. Moore and M. O. Scully, "Prospects for an X-Ray Free-Electron Laser", in Free-Electron Generators of Coherent Radiation, edited by C. A. Brau, S. F. Jacobs and M. O. Scully, Proc. SPIE 453, 393 (1984). It is reproduced in Appendix I.

III. NUCLEAR SPECTROSCOPY WITH X-RAY LASERS

One can reasonably expect that X-ray lasers when available, will have a strong impact on nuclear spectroscopy, similar to the impact that optical lasers have had on atomic spectroscopy. It is particularly interesting to anticipate the possibility of modulating nuclear beta decay or orbital electron capture rates by means of an X-ray laser. This possibility was analyzed for a simple case of a parent nucleus (Z,N) decaying into a daughter nucleus (Z ± 1, N ± 1), where the parent nucleus has just two states, a (lower) and b (upper), and the daughter nucleus has just one state c. Both a and b are assumed to be unstable with respect to weak decays. The X-ray laser frequency is assumed to be tunable to the level separations of a and b. The situations similar to the highly forbidden decay of ${}^{129}_{53}I$ to a low lying excited state of ${}^{129}_{54}Xe$. The physical picture is exactly analogous to Raman scattering. The results of the calculation show that the modulation of the decay rate is extremely sensitive to the tuning of the laser frequency to the a → b transition frequency, which has natural quantum limits arising from the spontaneous decay of the states, as well as from internal conversion processes. If one can achieve tuning on the order of $\Delta\omega/\omega \sim 10^{-13}$, powers on the order of 10^{14} W/cm^2 are needed in order to see observable effects in the decay of ${}^{129}_{53}I$. This calculation is described in the paper by W. Becker, R. R. Schlicher, M. O. Scully, M. S. Zubairy, and M.

Goldhaber, "Nuclear Spectroscopy with X-ray Lasers", Phys. Lett. 131B (1983) 16, which is reproduced in Appendix II.

One interesting result concerns scattering processes or decay of charged or neutral pointlike particles in a laser field idealized by a plane wave of arbitrary polarization. The wave functions of the charged particles are then Volkov solutions. In the paper by W. Becker, G. T. Moore, R. R. Schlicher and M. O. Scully, "A note on total cross sections and decay rates in the presence of a laser field", Phys. Lett. 94A (1983) 131, which is reproduced in Appendix III, it is shown that in the quasiclassical limit ($\hbar \to 0$), the differential cross section or decay rate can be represented as an integral with respect to time over an instantaneous cross section of decay rate which depends on the value of the laser field at a given time. Since this instantaneous rate is, in principle, a measurable quantity, it must depend on the canonical momentum $\vec{p}_i$ of the i^{th} particle and on the laser field $\vec{A}(t)$ only via the mechanical momentum $\vec{\pi}_i(t) = \vec{p}_i - e_i\vec{A}(t)$. If the differential rate is integrated over the final momenta in order to obtain the total cross section of the total decay rate, the dependence of the final rate on the laser field is completely eliminated by changing the integration variable from $\vec{p}_i$ to $\vec{p}_i - e_i\vec{A}(t)$. Consequently, the total decay rate of a neutral particle is unaffected by a laser field, and a charged particle, if stable in vacuum, remains stable in a laser field. The quasiclassical approximation breaks down if the strength of the external field becomes comparable with the critical field strength $E_{crit} = m^2c^3/e\hbar \sim 10^{16}$ Volt/cm, which is out of reach of present lasers or those anticipated in the near future.

The preceding considerations can be extended to nucleons bound in a self-consistent potential when the nucleon-laser field interaction is negligible. Given the energy of a typical laser quantum of 1 eV and a typical nuclear level difference of 1 MeV, this is an excellent approximation. In this case it can be shown again that, in the quasiclassical limit, total decay rates

are unaffected by laser fields which are weaker than the critical field. Recent claims that forbidden nuclear beta decay can be drastically enhanced by an intense radio frequency field are contested. This work is described in Appendix IV, which is a preprint entitled "A no-go theorem concerning the enhancement of nuclear decays by intense radiation fields", by W. Becker, R. R. Schlicher and M. O. Scully (submitted for publication in Phys. Lett.).

At the 1983 NATO Advanced Study Institute in Madrid, Spain, M. O. Scully gave seminars on the subject of allowed nuclear beta decay in intense laser fields. The notes for these lectures were prepared by R. R. Schlicher, W. Becker and M. O. Scully and are reproduced in Appendix IV. The notes cover topics such as total decay rates and differential electron energy distributions of nuclear beta decay in the presence of a laser field. The emphasis is on the classical versus quantum mechanical description, questions of gauge invariance, and the fact that the classical description is sufficient for a much wider range of parameters than one might initially assume.

APPENDIX I

Prospects for an X-Ray Free-Electron Laser

Julio Gea-Banacloche, Gerald T. Moore and Marlan O. Scully

Institute for Modern Optics, Department of Physics and Astronomy
University of New Mexico, Albuquerque, New Mexico 87131

Abstract

We present an overview of the regimes in which operation of an x-ray free-electron laser (FEL) may be feasible, including discussion of static and electromagnetic wigglers, quantum recoil, high-gain operation, mass-shift broadening, and electron beam quality.

Introduction

The free-electron laser is a swept-gain laser which offers unique possibilities for scaling to short-wavelength operation. Atomic charge-exchange swept-gain devices[1,2] which have been proposed for operation at x-ray wavelengths are limited by rapid spontaneous decay of the upper lasing level. By contrast, energy input to the FEL is in the stable form of electron kinetic energy, and the electrons themselves (not just the gain) propagate at nearly the speed of light. No atomic nuclei or bound electrons are present to complicate the physics (except in the resonator materials).

We shall consider here possible regimes for an x-ray FEL lasing at the fundamental frequency $\omega_s = 2c\gamma_s{}^2k_q$ or $\omega_s = 4c\gamma_s{}^2k_i$, where γ_s is the longitudinal Lorentz factor of the electron beam and k_q (k_i) is the wave vector of the static (electromagnetic) wiggler. Generation of higher harmonics[3,4] of ω_s is another important mechanism for producing short wavelengths, but will not be treated in this paper.

Because available resonators for x-rays are of poor quality, it is necessary to have large gain per pass to obtain lasing. If the gain is sufficiently large, one can dispense with the resonator altogether and lase by amplified spontaneous emission (ASE). It is not known how much gain is required for an FEL to lase by ASE. This mode of operation would have advantages, since the repetition rate of electron pulses (and wiggler pulses in the case of a counterpropagating electromagnetic wiggler field) could be arbitrarily low. There would also be no problem of resonator alignment. The coherence time would be limited to the slippage time $L/2\gamma_s{}^2c$, where L is the wiggler length. In other words, the coherence in terms of x-ray periods could not exceed the number of wiggler periods. Probably one could not do much better than this using a low-Q resonator.

We shall consider two types of wiggler, the uniform wiggler and the Gaussian beam. The uniform wiggler could be a conventional permanent-magnet or bifilar-helical type. It could also be a counterpropagating electromagnetic wave contained in a wave guide. The latter might be less expensive, particularly if one needs a very long wiggler. In either case the wavelength λ_q or $\lambda_i = 2\lambda_q$ is on the order of centimeters. To obtain x-rays from such a wiggler, the electron energy must be in the GeV range.

The Gaussian-beam wiggler uses a counterpropagating focused Gaussian beam from a high-power infrared laser as the wiggler field. Because λ_i is small, the electron energy need only be on the order of tens of MeV. The interaction length is limited both by the Rayleigh range Z_R and the infrared pulse length. Moreover, slowing of the electrons in the vicinity of the beam waist is the dominant source of homogeneous line broadening for a powerful (terawatt) wiggler field. Quantum recoil of the electrons is also typically important for this type of FEL.

Low-Gain Uniform Wiggler

Let us now consider the uniform wiggler in detail. The quantity of most interest is the small-signal gain. We first evaluate the gain per pass G assuming G << 1. Then we generalize to the large-gain regime G >> 1. For the sake of definiteness, we assume a static wiggler. We neglect pulse effects, assume the x-ray laser beam to be monochromatic, and account for the detuning of electron energies E from resonance by the usual detuning parameter ω, defined by

$$E = Mc^2\gamma_s(1 + \omega/2k_q) \quad . \tag{1}$$

Here $M = m\bar{\Delta}^{-\frac{1}{2}}$ is the electron mass multiplied by a mass-shift correction given by

$$\bar{\Delta} = 1 + (eB/mck_q)^2 \ , \tag{2}$$

where B is the RMS magnetic field. In practice we are interested in cases where $\bar{\Delta}$ is at most a few times unity.

If we postpone for now the question of emittance, there are three detuning widths governing the expression for the gain. First, there is the homogeneous transit-time broadening with bandwidth $2\pi/L$. Second, there is the inhomogeneous broadening U characterizing the width of the normalized electron energy distribution $f(\mu)$. Third, there is the quantum recoil 2q undergone by electrons when they emit an x-ray photon.

The scaling of U is different for linear and circular accelerators (or storage rings). For linear accelerators the spread δE is determined mainly in the initial part of the accelerator. With care one can accelerate the electrons to high energy without increasing δE. From Eq. (1) we see that

$$U = \frac{2k_q}{Mc^2\gamma_s}\,\delta E = \frac{(4\pi)2^{\frac{1}{2}}}{Mc^2}\,\delta E\ \lambda_s^{\frac{1}{2}}\ \lambda_q^{-\frac{3}{2}} \ . \tag{3}$$

The value of δE depends on the particular accelerator, but probably the minimum value one could obtain is[5]

$$\delta E = eI/4\pi\varepsilon_0 c = mc^2(I/17{,}000\ \mathrm{A}) \ , \tag{4}$$

where I is the electron current. This limit comes from the Coulomb repulsion of the electrons. The energy spread in the Stanford superconducting LINAC is about 100 times larger.

In a storage ring the dominant energy spread[6] is due to noise associated with synchrotron radiation and gets larger at higher energies. One has approximately

$$\delta E = (\hbar/2mc\rho)^{\frac{1}{2}}(E^2/mc^2) \ , \tag{5}$$

where ρ is the bending radius of the ring. Combining Eqs. (5) and (1) yields

$$U = 2\pi(\hbar\bar{\Delta}/mc\rho)^{\frac{1}{2}}\ \lambda_s^{-\frac{1}{2}}\ \lambda_q^{-\frac{1}{2}} \ . \tag{6}$$

The fact that Eq. (6) scales as $\lambda_s^{-\frac{1}{2}}$ suggests that a storage-ring x-ray FEL is impracticable if λ_s is too small, since the maximum L for homogeneously broadened operation will scale as $\lambda_s^{\frac{1}{2}}$. Our estimates indicate a minimum λ_s of about 60 Å. The storage ring seems an excellent option for λ_s of 100 Å or greater.[7] With a linear accelerator it may be possible to scale λ_s down to 5 Å or less.

The quantum recoil $\hbar\omega_s$ when expressed in detuning units becomes

$$2q = (8\pi^2\, 2^{\frac{1}{2}}\hbar/Mc)\,\lambda_s^{-\frac{1}{2}}\ \lambda_q^{-\frac{3}{2}} \ . \tag{7}$$

In the small-gain, small-signal regime the gain may be written as $G = C\Gamma$, where

$$C = \frac{e^3 I B^{-2}}{2^{\frac{1}{2}}\pi M^3 c^5 \varepsilon_0 \Sigma}\ \lambda_s^{\frac{3}{2}}\ \lambda_q^{-\frac{1}{2}} \ , \tag{8}$$

$$\Gamma = \frac{1}{2q}\int d\mu\,[f(\mu + q) - f(\mu - q)]\,|A(\mu)|^2 , \tag{9}$$

$$|A(\mu)|^2 = \left|\int_0^L dz\ e^{-i\mu z}\right|^2 = \left(\frac{\sin \mu L/2}{\mu/2}\right)^2 \ . \tag{10}$$

We have used the Doppler upshift condition to eliminate explicit reference to ω_s in Eq. (8). I is the current within the laser mode area Σ. The coefficient η is one for a helical wiggler and a well known[3] difference of Bessel functions for a linear wiggler. Equation (9)

expresses Γ as the difference between the forward and reverse quantum processes. This difference is replaced by a derivative with respect to u in classical treatments of the FEL. This replacement is sometimes, but not always, justified for the x-ray regime.

We may distinguish six regimes depending on the relative sizes of the homogeneous, inhomogeneous, and quantum widths, as shown in Table 1.

As an example, let us consider a helical wiggler with the parameters of the Stanford wiggler, except that it is longer, and use this wiggler to generate 5 Å radiation. The system parameters we use are shown in Table 2.

$2q < U < 2\pi/L$,	$\Gamma \propto L^3$
$2q < 2\pi/L < U$,	$\Gamma \propto L/U^2$
$U < 2q < 2\pi/L$,	$\Gamma \propto L^3$
$2\pi/L < 2q < U$,	$\Gamma \propto L/U^2$
$U < 2\pi/L < 2q$,	$\Gamma \propto L^2/q$
$2\pi/L < U < 2q$,	$\Gamma \propto L/qU$

Table 1

Quantity	Symbol	Value
Wiggler wavelength	λ_q	3.2 cm
Magnetic field	B	.24 T
Mass shift	$\bar{\Delta}$	1.512
Laser wavelength	λ_s	5 Å
Electron energy	E	3.554 GeV
Current	I	10 A
Electron beam power	EI/e	3.564×10^{10} W
Laser mode area	Σ	.16 mm^2

Table 2

With these parameters we calculate a homogeneously broadened gain of unity at L = 81 m. If we take the inhomogeneous broadening to be given by Eqs. (3) and (4), and we use Eq. (7) for the quantum recoil, we calculate $2\pi/U = 1.9 \times 10^5$m and $\pi/q = 2.3 \times 10^4$m. These correspond to the wiggler lengths at which the homogeneous broadening would equal the inhomogeneous broadening or the quantum recoil respectively. We conclude that δE could be more than three orders of magnitude larger than the value given by Eq. (4) before inhomogeneous broadening begins to affect the gain.

So far we have merely assumed that λ_q = 3.2 cm without justifying whether this is close to an optimal value. We can get a more global picture of the situation on a log-log plot of L versus λ_q, as shown in Fig. 1. Here the point of unit gain at λ_q = 3.2 cm, L = 81 m is marked by an X. The two lines with slope 3/2 indicate the transitions where $\pi/L = q$ and $2\pi/L = U$. If we hold B and Σ constant, and neglect minor effects due to mass shift, we can generate a curve along which G = 1 by using Eq. (8) and Table 1. This gives the three-segment solid curve in Fig. 1. Alternatively, if we assume that the mode area Σ is governed by diffraction of the x-rays, then $\Sigma \propto \lambda_s L$ and we generate the dotted curve in Fig. 1. We can generate curves along which the gain has other values by parallel transport of these curves along the transition lines. These curves of constant gain are invalid if λ_q becomes too large. This is because the mass-shift corrections then become substantial and because the wiggle amplitude may exceed the assumed mode size.

We can infer from Fig. 1 that it is best to choose conditions close to the transition lines, since L is minimized there. The choice λ_q = 3.2 cm gives us some latitude if δE is larger or if we want to increase L further to get more gain. We also see that it is very disadvantageous to decrease λ_q to operate in the inhomogeneously broadened regime. The main point in trying to use a terawatt infrared laser as a wiggler is simply that the magnetic field one can generate is much larger than in a conventional wiggler.

High-Gain Uniform Wiggler

The classical cold-beam small-signal equations for the FEL have been published previously.[5] The extension of these equations to include inhomogeneous broadening is straightforward. The extension to include quantum recoil is too lengthy to derive here, but can be done by analyzing the coupled Maxwell and Klein-Gordon equations. The result, itself, is very simple. In the cold-beam limit the equations take the form

$$dE_s/dz = i \frac{eIB\lambda_q \eta}{4\pi\epsilon_0 Mc^2 \gamma_s \Sigma} e^{-i\mu_0 z} K_2 , \tag{11}$$

$$\frac{dK_2}{dz} = K_1 , \tag{12}$$

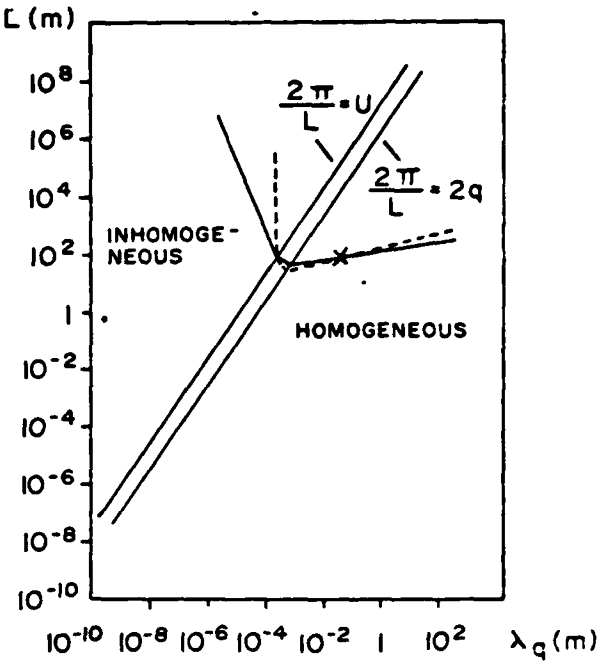


Figure 1. Curves of constant gain on a log-log plot of wiggler length L versus wiggler wavelength λ_q provide a convenient global picture of regions where operation of a short-wavelength FEL may be possible. In this figure we fix the laser wavelength, the wiggler magnetic field, and the current at the values given in Table 2. The point marked X corresponds to the unit-gain example discussed in the text. The energy spread is held constant at 300eV, independent of the electron energy. The quantum recoil in this example exceeds the inhomogeneous broadening, and the two parallel lines of slope 3/2 indicate the transitions where these respective widths equal the homogeneous broadening. Using Eq. (8) and Table 1, and assuming the laser mode area Σ is constant, we construct the three-segment solid curve along which G = 1. The slopes of the three segments (from right to left) are 1/6, -1/2 and -5/2. By parallel transport of this curve along the transition lines, we can generate curves along which G has other values. If G is small, then $G \propto \lambda_q^4$ at the transitions. The value of G on a given curve scales as $B^2 I/\Sigma$. We see that it is optimal (in terms of minimizing L) to operate near the transition lines. Also, it is disadvantageous to choose λ_q too small, unless one can compensate by increasing the wiggler field. The dotted curve is generated by taking $\Sigma \propto \lambda_s L$, as is appropriate if the mode area is limited by diffraction of the x-ray beam. In this case the segments of the curves of constant gain have slopes 1/4, -1 and $-\infty$, and the gain along the transition lines scales as $\lambda_q^{5/2}$.

$$\frac{dK_1}{dz} = \frac{e^2 B\eta}{M^2 c^3 \gamma_s^2}\, e^{i\mu_0 z} E_s - q^2 K_2 \ . \tag{13}$$

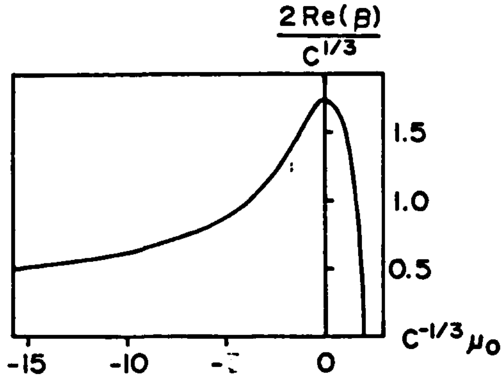
Here E_s, K_2, and K_1 are the laser field, the density-bunching amplitude, and the velocity-bunching amplitude. The last term in Eq. (13) is the quantum correction, and μ_0 is the velocity detuning from resonance. The quantum correction arises from an interference between probability amplitudes with and without the photon emission. In the small-gain limit Eqs. (11)-(13) can be used to derive (8)-(10) in the case $f(\mu) = \delta(\mu - \mu_0)$. However, in general the solution of Eqs. (11)-(13) consists of three exponential modes having the spatial dependence exp (βz), where β obeys the cubic dispersion relation

$$\beta[(\beta + i\mu_0 + U)^2 + q^2] = iC \ . \tag{14}$$

In Eq. (14) we have generalized to the case where $f(\mu)$ is not a δ-function, but is a Lorentzian of width U (HWHM) centered at $\mu = \mu_0$. At most one of the roots of Eq. (14) has a positive real part, and the corresponding mode will dominate near the end of a sufficiently long wiggler. At the entrance of the wiggler the three modes contribute about equally in order to satisfy the initial conditions $K_1(0) = K_2(0) = 0$. Figure 2 shows the growth rate 2 Re(β) for q = 0, U = 0 as a function of μ_0. We see that maximum gain is obtained on resonance, where

$$G \approx \frac{1}{9}\exp(3^{1/2} C^{1/3} L) \ . \tag{15}$$

For the parameters of Table 2 we calculate a gain per pass of 1000 if L = 276 m. This might be enough to lase without a resonator. A conventional wiggler of this length would be very expensive, but one might be able to instead use a microwave confined by a waveguide.[7] The microwave power required would be about 1 GW, and the minimum microwave pulse length needed would be 2L = 552 m, so that the microwave pulse energy would be about 1.8 kJ. Such microwave powers are obtainable and can be propagated in a single wave-guide mode. Microwave pulse lengths obtained so far at this power are a bit short of what we would like. Note that the slippage time $L/2\gamma_s^2 c$ is only 1.44×10^{-14} sec.



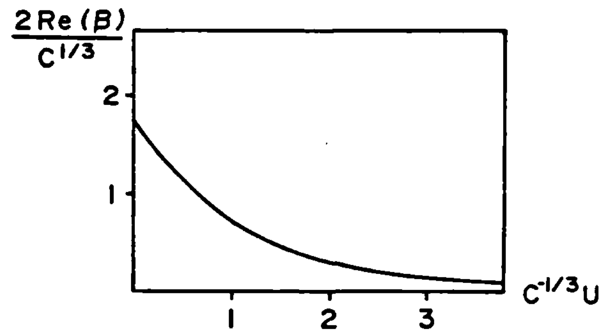


Figure 2. The gain coefficient $2\,\mathrm{Re}(\beta)$ for the exponentially growing mode is shown as a function of velocity detuning μ_0, neglecting quantum recoil and inhomogeneous broadening.

Figure 3. The gain coefficient $2\,\mathrm{Re}(\beta)$ is shown as a function of inhomogeneous broadening U for the optimal detuning $\mu_0 = U/3^{1/2}$. Quantum recoil is neglected.

The effect of a Lorentzian inhomogeneous broadening of width U (again neglecting q) is to shift the optimal detuning to $\mu_0 = 3^{-1/2}U$ and to reduce the gain, as shown in Fig. 3.

In Fig. 4 we show $2\,\mathrm{Re}(\beta)$ for the case where the inhomogeneous broadening U is set equal to zero, but the quantum recoil q is taken into account. If $C \lesssim q^3$, then the gain is peaked near $\mu_0 = q$. If C becomes larger than q^3, then $2\,\mathrm{Re}(\beta)$ approaches the classical result given in Fig. 2, but continues to have a sharp cut-off on the negative, as well as positive, end of the detuning range. The quantum recoil reduces the maximum of $2\,\mathrm{Re}(\beta)$ below its classical value. For the parameters of Table 2 this is a very slight effect, since $C/q^3 = 2.7 \times 10^6$.

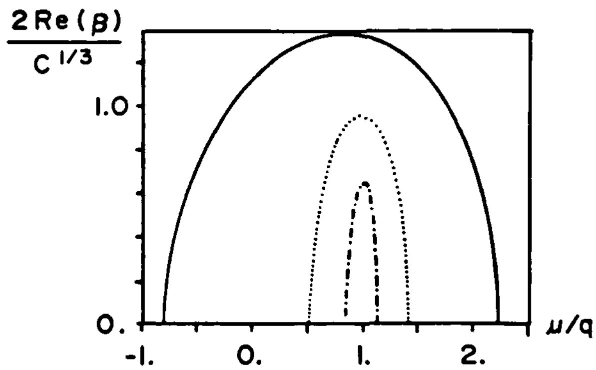


Figure 4. The gain exponent (in units of $C^{1/3}$) vs. the detuning (in units of q) for several cases in the quantum-mechanical regime. Solid Line: $C/q^3 = 1$. Dashed line: $C/q^3 = .1$. Dash-dot line: $C/q^3 = .01$.

We expect the FEL to saturate when the density bunching amplitude $K_2(z)$ becomes of the order of unity. From this condition we can derive the saturation power

$$P_{sat} = \left(\frac{I^2 B \lambda_q^{\,2}}{16\pi^2}\right)^{2/3} \left(\frac{c}{\varepsilon_0 \Sigma}\right)^{1/3} \quad . \tag{16}$$

For the above numerical example we calculate $P_{sat} = 1.7$ MW.

Gaussian-Beam Wiggler

Infrared lasers such as CO_2 or Nd-glass can generate very large electromagnetic fields, which suggests that we consider high-power pulses from such lasers as wiggler fields for an x-ray FEL. Since λ_q is short, one needs relatively low electron energies (tens of MeV). The interaction length is limited to half the pulse length of the infrared (pump) laser, since the electrons and pump field are counterpropagating, each at nearly the speed of light. Moreover, the interaction length is limited to a value on the order of the Rayleigh range Z_R by diffractive spreading of the pump field. In the present analysis we take the pulse length infinite for simplicity and only take into account the diffractive spreading. Then the on-axis pump field is of the form

$$\hat{A}_q(z,t) = \frac{1}{1 - iz/Z_R} A_i \exp[-i\omega_i(t + z/c)] \ . \tag{17}$$

The wiggler field is therefore <u>tapered</u>. The electrons will be slowed up in the vicinity of the focus, where more of their energy gets transferred into transverse oscillation. This results in greater homogeneous broadening than for a uniform wiggler. Electrons within the homogeneous bandwidth will in general be resonant (move at the same speed as the ponderomotive potential) only at one point entering and one point leaving the focal region. The electron slowing is a consequence of the variable mass shift. Even though $\Sigma - 1$ will normally be very small for the Gaussian-beam wiggler, the cumulative effect over the many wiggler wavelengths within the interaction region can be large. It turns out that the size of the effect is characterized by a parameter ε depending only on the power P_i in the pump field,

$$\varepsilon = (2e^2/\pi\varepsilon_0 m^2 c^5) P_i \ . \tag{18}$$

For a terawatt beam this gives $\varepsilon = 913$. Let us specify that the energy-detuning μ be zero for electrons which at $z = \pm\infty$ travel at the speed $c(k_s - k_i)/(k_s + k_i)$ of the ponderomotive potential. Then the homogeneous bandwidth for large ε is approximately $0 < \mu Z_R < \varepsilon$. The function $\Lambda(\mu)$ in Eq. (10) is replaced by

$$\Lambda(\mu) = \int_{-\infty}^{\infty} dz \, \frac{1}{1 - iz/Z_R} \exp[-i\mu z + i\varepsilon \arctan(z/Z_R)] \ . \tag{19}$$

Also, the constant C in Eq. (8) can be written in terms of ε as

$$C = \frac{eI\ \varepsilon}{2c^3 \varepsilon_0 \Sigma m \gamma_s{}^3 Z_R} \ . \tag{20}$$

The two points of stationary phase in Eq. (19) lie at $z = \pm Z_R(\varepsilon/\mu Z_R - 1)^{1/2}$, and Λ is approximately a sum of contributions from these two points. The interference of these two contributions makes $|\Lambda(\mu)|^2$ a rapidly oscillating function of μ if ε is large. A graph of this function for $\varepsilon = 92$ is shown in Fig. 5. The width of the large peak on the right side is approximately $2\varepsilon^{1/3}/Z_R$. It is possible to express $\Lambda(\mu)$ exactly in terms of Whittaker functions.[11] An approximate expression in terms of Airy functions which is valid near $\mu = \varepsilon/Z_R$ (that is, around the main peak on the right side) is

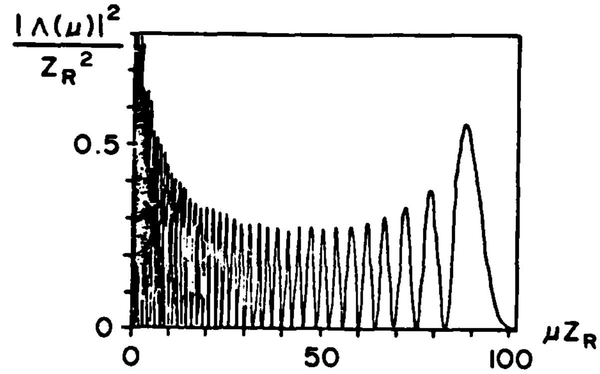


Figure 5. The mass-shift broadened line-shape $\Gamma(\mu)$ as a function of energy detuning. THe main peak on the right (centered at about $\mu = \varepsilon/Z_R$) corresponds to a detuning such that particles are resonant near the focus of the Gaussian beam.

$$|\Lambda(\mu)|^2 \approx 4\pi^2 \frac{Z_R{}^2}{\varepsilon^{2/3}} \mathrm{Ai}^2 \left(\frac{\mu Z_R - \varepsilon}{\varepsilon^{1/3}} \right) \ , \tag{21}$$

where Ai(x) is the Airy function.

As μ becomes less than ε/Z_R, the function $|\Lambda(\mu)|^2$ becomes more and more rapidly oscillating. In cases where the inhomogeneous broadening is large compared to the period of these oscillations, we can average over the oscillations to obtain (in the stationary-phase approximation)

$$|A(\mu)|^2 = (2\pi Z_R{}^2/\epsilon)\left[\frac{\mu Z_R}{\epsilon}\left(1 - \frac{\mu Z_R}{\epsilon}\right)\right]^{-1/2} . \quad (22)$$

Contributions to $|A(\mu)|^2$ near $\mu = 0$ come from z points far away from the beam waist. In practice such contributions are excluded by the finite pump pulse length.

Clearly there are a number of possible operating regimes, depending on the relative sizes of U and 2q, compared to $2\epsilon^{1/3}/Z_R$ and ϵ/Z_R. The most favorable case is one in which all the electron energies are contained within the main peak on the right of the gain curve (see Fig. 5), and where the Compton recoil is of the same order as the width of that peak, so that the absorption process is well separated from the emission process (with no electrons contributing to the former). In other words, we require

$$U < \frac{2\epsilon^{1/3}}{Z_R} = 2q , \quad (23)$$

where for an electromagnetic pump pulse of wavelength λ_i, 2q is given by

$$2q = \frac{32\pi^2\hbar}{mc} \lambda_s{}^{-1/2} \lambda_i{}^{-3/2} \quad (24)$$

(This corresponds to Eq. (7) with $\lambda_i = 2\lambda_q$). Under these conditions, using Eq. (21), we have

$$G = \frac{C}{2q} 4\pi^2 \frac{Z_R{}^2}{\epsilon^{2/3}} Ai^2 \left(\frac{\mu Z_R - \epsilon}{\epsilon^{1/3}}\right) \quad (25)$$

and

$$G_{max} = \frac{eI\epsilon^{1/3} \lambda_s{}^2 Z_R}{2c^2\epsilon_0\Sigma\hbar} \times 0.29 , \quad (26)$$

where we used Eq. (20) and the fact that the maximum value of $Ai^2(x)$ is 0.29.

Once the wavelength of the pump is chosen, the value of the ratio $\epsilon^{1/3}/Z_R$ is limited by the condition (23). Recall that ϵ is proportional to the power in the pump pulse and Z_R is proportional to the pulse length. Increasing Z_R helps to increase the gain, according to Eq. (26), but it also reduces the width of the gain peak, and eventually one ends up in the inhomogeneously broadened regime.

A serious limitation to the minimum achievable energy spread comes from Eq. (4), which gives the difference in the kinetic energy of the electrons at the edge and the electrons on the axis of an unneutralized beam. The inhomogeneous width resulting from this effect is given by

$$U = 16\pi \lambda_s^{1/2} \lambda_i^{-3/2} (I/17{,}000 \text{ A}) . \quad (27)$$

Note that I here is not necessarily the total current, but only the current through the laser mode area Σ. It seems as if one could make this very small by making the laser mode area very small (hence decreasing I while keeping I/Σ constant). In reality one is limited here too by the fact that the electrons have some transverse velocity and if Σ is very small, they might drift out of the interaction volume and contribute little to the gain. A measure of this transverse velocity spread is provided by the emittance of the beam.

Without dwelling on the details, we present in Table 3 some optimal values for two possible cases: one in which the pump pulse is a 10^{12} W pulse from a Nd:YAG laser at 1.1 μm, and one in which it is a 10^{11} W pulse from a CO_2 laser at 10.6 μm. In both cases we assume operation at $\lambda_s = 5.7$ Å. We have chosen this wavelength because of the possibility at this wavelength of building a reasonably good resonator using Bragg diffraction from Ge crystals.[10] We take the length of the pump pulse to be roughly equal to the Rayleigh length to obtain efficient coupling.

The CO_2-laser-pumped case appears to be more favorable, except that the relative energy spread is rather small. The absolute energy spread, however, is the same in both cases, and

Pump Parameters		
Pump wavelength λ_i	1.1 μm	10.6 μm
Pump power P_i	10^{12} watts (ϵ = 913)	10^{11} watts (ϵ = 91.3)
Rayleigh length Z_R	4.4 mm	6 cm
Total pulse energy $P_i(2Z_R/c)$	30 Joules	40 Joules
Electron Beam Parameters		
Electron beam energy E	11 MeV	34 MeV
Normalized emittance	5.2×10^{-6} m rad	same
Peak current density I/Σ	10 MA/cm^2	1.6 MA/cm^2
Peak current (total)	9 kA	4.7 kA
Energy spread $\delta E/E$	2×10^{-4}	6×10^{-5}
Electron beam cross section	9 mm^2	29 mm^2
Laser mode area	7×10^{-10} m^2	4.4×10^{-9} m^2
Gain	0.4	0.4

Table 3

equals 2.4 keV (equal to the Compton recoil at the wavelength considered). The normalized emittance has been chosen equal to that of the Stanford FEL experiment. A smaller emittance would help lower the requirements on the total current. Note in this respect that the total peak current across the laser mode area is in both cases only about 70 A, so that many electrons are being "wasted". As mentioned before, this is because one needs to keep the electron beam area large so that it won't spread too fast, given the emittance we have assumed. The need for high current densities, on the other hand, seems unavoidable, as may be seen from Eq. (26).

The situation is, as one might expect, more favorable for longer λ_s. As a final example, we give some figures for operation at 100 Å, using again a terawatt pulse from a Nd-YAG laser. In this case, we increase Z_R over the value given in the previous example to satisfy Eq. (23). With the value Z_R = 1.74 cm (corresponding to a total pump pulse energy of 120 J) one gets a gain of unity (100%) for a current density of 21 kA/cm^2 and a peak current of 190 A. The electron energy is only 2.6 MeV in this example, and the maximum allowable energy spread to stay in the homogeneously broadened regime is $\delta E/E \approx 5 \times 10^{-5}$ (δE = 0.1 keV).

Discussion

In this paper we have had space only to describe the basic physics of various regimes for possible operation of an x-ray FEL. We have seen that substantial gain can be achieved. However, clearly much work remains in areas such as accelerator design, electron beam transport, x-ray resonators, and optics for injection and recirculation of high-power electromagnetic wiggler pulses. The subject is relatively undeveloped, and additional regimes of operation and mechanisms for gain enhancement may remain to be discovered.

We conclude by assessing several items of concern to the designer of an x-ray FEL. Diffractive spreading of the x-ray beam is a minor effect, although appreciable for a very long system. The value of Σ which we used for the long-wiggler example is consistent with the effect of diffractive spreading. For the Gaussian-beam wiggler one can have a very small Σ. However, if the electron beam area is much larger than Σ, it could be hard experimentally to observe the lasing against a strong background of incoherent emission from all the electrons. Because of the high loss of x-ray resonators, one can expect the laser output itself to be rather incoherent.

Absolute tolerances on the long conventional (or microwave) wiggler are about the same as in present-day conventional wigglers. For the Gaussian-beam wiggler the pump beam must be coherent over the interaction length. Effects due to the finite pulse length of the wiggler pulse may also be important, and remain to be calculated. If one operates with $\epsilon \gg 1$, then either the power in the pump pulse must be kept nearly the same on each pass or the electron energy must vary from pass to pass in order to maintain the gain at constant x-ray wavelength.

Transverse variations in the wiggler field are a serious concern for all the regimes we have considered, since they can lead to de-phasing of the electrons with respect to the ponderomotive potential. We estimate that such effects can be adequately controlled by keeping the e-beam area small enough that the electrons essentially see the on-axis wiggler field.

Emittance can also reduce the gain. We estimate that a normalized emittance equal to or less than that of the Stanford superconducting accelerator is sufficient to prevent serious gain depletion.

One reason why it is interesting to study the physics of the x-ray FEL is that features of the problem which are of minor importance at long wavelengths here become dominant. In particular, quantum and other fluctuations, as well as the development of coherence in three dimensions through ASE, are dominant processes. Future theoretical work, together with experiments at short wavelengths, should lead to a better understanding of this largely uncharted territory.

Acknowledgments

This research was supported by the Office of Naval Research and the Air Force Office of Scientific Research. We profited by discussions with P. Dobiasch and P. Meystre of the Max-Planck Institute for Quantum Optics.

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APPENDIX II

NUCLEAR SPECTROSCOPY WITH X-RAY LASERS

W. BECKER, R.R. SCHLICHER, M.O. SCULLY, M.S. ZUBAIRY
Institute for Modern Optics [1], *Department of Physics and Astronomy, University of New Mexico, Albuquerque, NM 87131, USA*
and Max-Planck-Institut für Quantenoptik, D-8046 Garching, West Germany

and

M. GOLDHABER
Brookhaven National Laboratory [2], *Upton, NY 11973, USA*

Received 1 June 1983

The effects expected from irradiating a beta-unstable nucleus having a very low-lying excited state with a resonant X-ray laser are investigated.

Much as the availability of optical lasers brought about a revolution in atomic spectroscopy, strong short wave length X-ray or gamma-lasers would induce a comparable breakthrough in nuclear spectroscopy. In this note we want to determine the laser specifications which would be necessary for that purpose. As an example we shall treat a situation as depicted in fig. 1. A parent nucleus (Z, N) in its ground state a is assumed to decay via beta$^{\mp}$-decay or orbital electron capture to the state c of the daughter nucleus $(Z \pm 1, N \mp 1)$ which can be the ground state or an excited state. In the presence of an incident laser field which is closely resonant with the excitation energy of an excited level b of the parent nucleus the decay can proceed alternatively via absorption of one laser photon and subsequent beta-decay from the level b. In the latter case the beta-decay is governed by the weak interaction matrix element V_{bc} instead of V_{ac} which specifies the ordinary beta-decay. By comparing the decays with and without the laser field, information can be obtained about the matrix element V_{bc} which is inaccessible otherwise. In particular, if the direct decay from a to c is forbidden whereas the decay from b to c is allowed, a large enhancement of the total decay can be achieved. A similar situation has been referred to as excited state beta-decay in astrophysics [1]. However, in the case we are considering, the decay proceeds virtually via the intermediate state b, whereas at temperatures considered in astrophysics the intermediate state is thermally populated. Also, a similar level scheme has been suggested in order to populate the upper level (level b in our notation) of a possible nuclear X-ray laser [2].

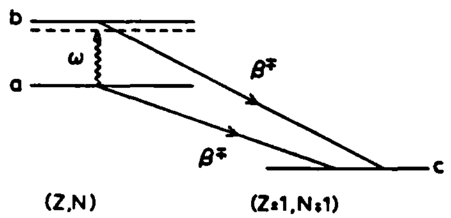

Fig. 1. The nuclear energy-level configuration, which is envisioned in the present paper; the parent nucleus (Z, N) with ground state a has a low-lying excited state b which is almost resonant with an incident X-ray laser of frequency ω. Both states a and b are beta-unstable.

Our formal approach is exactly analogous to the treatment of Raman scattering. We assume that the laser is almost resonant with the energy separation from a to b. The Schrödinger equation then yields a system of equations for the amplitudes of the levels a, b and c. In the interaction representation it reads [3]

[1] Supported in part by the Air Force Office of Scientific Research under grant No. AFOSR-81-0128.
[2] Supported by the US Department of Energy under Contract No. DE-AC02-76CH00016.

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$$\mathrm{i}\dot{a} = V^*_{ab}\exp[-\mathrm{i}(\epsilon_{ba}-\omega)t]b + V^*_{ac}\exp[\mathrm{i}(\epsilon_{ac}-\epsilon)t]c\,, \tag{1a}$$

$$\mathrm{i}\dot{b} = V_{ab}\exp[\mathrm{i}(\epsilon_{ba}-\omega)t]a + V_{cb}\exp[\mathrm{i}(\epsilon_{bc}-\epsilon)t]c\,, \tag{1b}$$

$$\mathrm{i}\dot{c} = V^*_{cb}\exp[-\mathrm{i}(\epsilon_{bc}-\epsilon)t]b + V_{ac}\exp[-\mathrm{i}(\epsilon_{ac}-\epsilon)t]a\,. \tag{1c}$$

Here the rotating wave approximation has been introduced, ω denotes the laser frequency, $\hbar\epsilon_{ba}$ is the excitation energy of the nucleus (Z, N), $\hbar\epsilon_{ac} = M(Z, N) - M(Z \pm 1, N \mp 1)$ is the difference of the *nuclear* masses and $\epsilon_{bc} = \epsilon_{ac} + \epsilon_{ba}$. In the case of beta$^\pm$-decay $\hbar\epsilon$ denotes the sum of the energies of the outgoing neutrino and electron or positron, including the rest mass. For orbital electron capture $\hbar\epsilon$ includes the energy of the emitted neutrino and the atomic binding energy difference of the captured electron minus the electron rest mass. At the intensities we need for the examples at the end of this letter we assume that the nucleus is not completely ionized so that electron capture can still take place. We also assume that internal conversion from b to a or the inverse process of nuclear excitation by electron transition [4] are negligible with respect to the stimulated electromagnetic transition. $\hbar V_{ab}$ is then a definite electromagnetic multipole matrix element or the sum of several of them, which can be related to the half-life of the level b. $\hbar V_{cb}$ and $\hbar V_{ac}$ are the weak interaction matrix elements which govern the decays from a and b to c, respectively. These matrix elements include not only nuclear wave functions but also the radial part of the electron and neutrino state. We do not have to include phenomenological dampings for the levels a and b, since our description includes all possible decays of them. If c is an excited state we should, in principle, add a damping term. Since however, the population of this level will remain small in any event, we can safely neglect this small amount of leakage out of the system.

Since due to the stimulating laser field the coupling between a and b can be quite strong we shall proceed by first solving eqs. (1a) and (1b) exactly after putting $c = 0$ and then integrating eq. (1c) after inserting the solutions for a and b. With the initial condition $a(0) = 1$, $b(0) = 0$ we get after the first step

$$a = (1/2\Lambda)[-\lambda_- \exp(\mathrm{i}\lambda_+ t) + \lambda_+ \exp(\mathrm{i}\lambda_- t)]\,, \tag{2a}$$

$$b = (V_{ab}/2\Lambda)[\exp(-\mathrm{i}\lambda_+ t) - \exp(-\mathrm{i}\lambda_- t)]\,, \tag{2b}$$

with

$$\Lambda = [(\epsilon_{ba}-\omega)^2/4 + |V_{ab}|^2]^{1/2}\,, \tag{3}$$

$$\lambda_\pm = (\omega - \epsilon_{ba})/2 \pm \Lambda\,. \tag{4}$$

Inserting eqs. (2a), (2b) we can now integrate eq. (1c) subject to the initial condition $c(0) = 0$. In the limit $t \to \infty$ we obtain for the transition probability per unit time

$$|c|^2/t = (\pi/2\Lambda^2) \times \{|V_{ab}|^2|V_{bc}|^2[\delta(\epsilon_{bc}-\epsilon+\lambda_+) + \delta(\epsilon_{bc}-\epsilon+\lambda_-)] + |V_{ac}|^2[\lambda_-^2\delta(\epsilon_{ac}-\epsilon-\lambda_+) + \lambda_+^2\delta(\epsilon_{ac}-\epsilon-\lambda_-)]\}\,. \tag{5}$$

We integrate eq. (5) over the phase space of the neutrino and the electron or positron, respectively, for beta$^\pm$-decay

$$\int \frac{\mathrm{d}^3p_e\,\mathrm{d}^3p_\nu}{(2\pi\hbar)^6} = \frac{1}{4\pi^4c^6}\int \mathrm{d}\epsilon_e\,\mathrm{d}\epsilon_\nu[\epsilon_e^2 - (mc^2/\hbar)^2]^{1/2}\epsilon_e\epsilon_\nu^2\,,$$

or just over the neutrino phase space for electron capture

$$\int \frac{\mathrm{d}^3p_\nu}{(2\pi\hbar)^3} = \frac{1}{2\pi^2c^3}\int \mathrm{d}\epsilon_\nu\epsilon_\nu^2$$

with $\epsilon_e = E_e/\hbar$, $\epsilon_\nu = E_\nu/\hbar$ in order to obtain the total transition rate per unit time. We find for beta$^\pm$-processes

$$\Gamma = \frac{1}{8\pi^3\Lambda^2c^6}\int \mathrm{d}\epsilon_e\epsilon_e[\epsilon_e^2 - (mc^2/\hbar)^2]^{1/2} \times \{|V_{ac}|^2[\lambda_-^2(\epsilon_{ac}-\epsilon_e-\lambda_+)_+^2 + \lambda_+^2(\epsilon_{ac}-\epsilon_e-\lambda_-)_+^2] + |V_{ab}|^2|V_{bc}|^2[(\epsilon_{ba}-\epsilon_e+\lambda_+)_+^2 + (\epsilon_{bc}-\epsilon_e+\lambda_-^2)_+]\}\,, \tag{6a}$$

and for electron capture

$$\Gamma = (4\pi\Lambda^2c^3)^{-1}\{|V_{ac}|^2[\lambda_-^2(\epsilon_{ac}+\epsilon_e-\lambda_+)_+^2 + \lambda_+^2(\epsilon_{ac}+\epsilon_e-\lambda_-)_+^2] + |V_{ab}|^2|V_{bc}|^2[(\epsilon_{bc}+\epsilon_e+\lambda_+)_+^2 + (\epsilon_{bc}+\epsilon_e+\lambda_-)_+^2]\}\,. \tag{6b}$$

where $x_+^2 = x^2$ for $x > 0$ and $x_+^2 = 0$ else. The electron energy for capture is given by $\epsilon_e = mc^2/\hbar - \epsilon$, where $\hbar\epsilon_b$ denotes the atomic binding energy. The first term in eqs. (6a), (6b) describes the direct decay from a to c, whereas the second specifies the decay via the virtual intermediate state which can be significantly enhanced if the incident photon is resonant.

Two limits of eq. (6a) are of interest: (a) if the interaction V_{ab} is weak or, equivalently, far off resonance, i.e., $|V_{ab}| \ll \frac{1}{2}|\epsilon_{ba} - \omega|$, we have for beta$^{\pm}$-decay (analogous equations hold for electron capture)

$$\Gamma = \frac{1}{2\pi^3 c^6} \int d\epsilon_e \epsilon_e [\epsilon_e^2 - (mc^2/\hbar)^2]^{1/2} \times \{|V_{ac}|^2 (\epsilon_{ac} - \epsilon_e)_+^2 + [|V_{cb}|^2 |V_{ab}|^2 / (\epsilon_{ba} - \omega)^2] \times [(\epsilon_{bc} - \epsilon_e)_+^2 + (\epsilon_{ac} + \omega - \epsilon_e)_+^2]\}, \qquad (7)$$

where the first term specifies the decay rate in the absence of the laser; (b) in the opposite case of comparatively small detuning, i.e. $\frac{1}{2}|\epsilon_{ba} - \omega| \ll V$, we have

$$\Gamma = \frac{1}{8\pi^3 c^6} \int d\epsilon_e \epsilon_e [\epsilon_e^2 - (mc^2/\hbar)^2]^{1/2} \times \{|V_{ac}|^2 [(\epsilon_{ac} - \epsilon_e + |V_{ab}|)_+^2 + (\epsilon_{ac} - \epsilon_e - |V_{ab}|)_+^2] + |V_{bc}|^2 [(\epsilon_{bc} - \epsilon_e + |V_{ab}|)_+^2 + (\epsilon_{bc} - \epsilon_e - |V_{ab}|)_+^2]\}. \qquad (8)$$

In this case, the two level system (a, b) is saturated. Hence the levels a and b are approximately equally populated, the matrix element V_{ab} has cancelled from eq. (8), and the direct decay from a to c is reduced by a factor of ~2 with respect to eq. (7) (note that the term $|V_{ab}|$ in the square bracket will be negligible under almost all circumstances).

Obviously, case (b) is of paramount interest since it allows for a comparison of the weak matrix elements V_{ac} and V_{bc}. The branching ratio of the two processes $a \to b \to c$ and $a \to c$ is proportional to $|V_{bc}|^2/|V_{ac}|^2$. The decay enhancement due to the laser reads in the particular case where V_{ac} is classified as forbidden whereas V_{bc} is allowed,

$$R_{sat} = \frac{|V_{bc}|^2}{2|V_{ac}|^2} \frac{(\text{phase space})_{bc}}{(\text{phase space})_{ac}}. \qquad (9)$$

In addition to the first factor in this ratio, also the second can be much larger than one, if $\epsilon_{bc} \simeq \epsilon_{ac} + \omega > \epsilon_{ac}$. For allowed beta-decays the phase space is given by the shape factor ($\hbar\epsilon_0$ denotes the maximum released energy) [5]

$$f(\epsilon_0) = \int_{mc^2/\hbar}^{\epsilon_0} d\epsilon\, \rho(Z, R) \times \epsilon[\epsilon^2 - (mc^2/\hbar)^2]^{1/2} (\epsilon_0 - \epsilon)^2 .$$

$\rho(Z, R)$ is the electron density at the nucleus which, if relativistic Coulomb corrections are taken into account, depends also on the nuclear charge and radius.

Case (b) applies when

$$K^2 = 4|V_{ab}|^2/(\epsilon_{ba} - \omega)^2 \gg 1 . \qquad (10)$$

In any experiment the laser will be tuned to resonance as far as possible. Hence, we should in the definition of K replace $|\epsilon_{ba} - \omega|$ by a quantity $\Delta\omega$, which is the bandwidth of the laser or the width of the nuclear level b, whichever is larger. The matrix element $|V_{ab}|$ is related to the spontaneous multipole matrix element $|V_{ab}^0|$ by

$$|V_{ab}| = N|V_{ab}^0| , \qquad (11)$$

where N is the number of laser photons, and V_{ab}^0 determines the spontaneous half life of the level b via [6]

$$|V_{ab}^0|^2 = (8\pi^2\omega^2 t_{1/2} V)^{-1} c^3 \ln 2 , \qquad (12)$$

with V the normalization volume. Here we assumed again that internal conversion be negligible. If it is not, the lifetime $t_{1/2}$ should be corrected for according to the tabulated internal conversion coefficients [6]. Using eqs. (11) and (12) we now have

$$K^2 = (2\pi^2)^{-1} c^3 \ln 2 (N/V)(\omega^2 \Delta\omega^2 t_{1/2})^{-1} , \qquad (13a)$$

or in terms of the intensity of the laser field

$$K^2 = 1.1 \times 10^{-42} (mc^2/\hbar\omega)^6 (\omega/\Delta\omega)^2 I [\text{W/cm}^2] \times (t_{1/2}[\text{ns}])^{-1} . \qquad (13b)$$

The requirements which have to be met by the laser in order to achieve $K^2 \geq 1$ are very severe.

However, even if $K^2 \ll 1$, eq. (7) shows that the decay is still enhanced by a factor of $1 + R$ with

$$R = K^2 R_{sat}\,, \qquad (14)$$

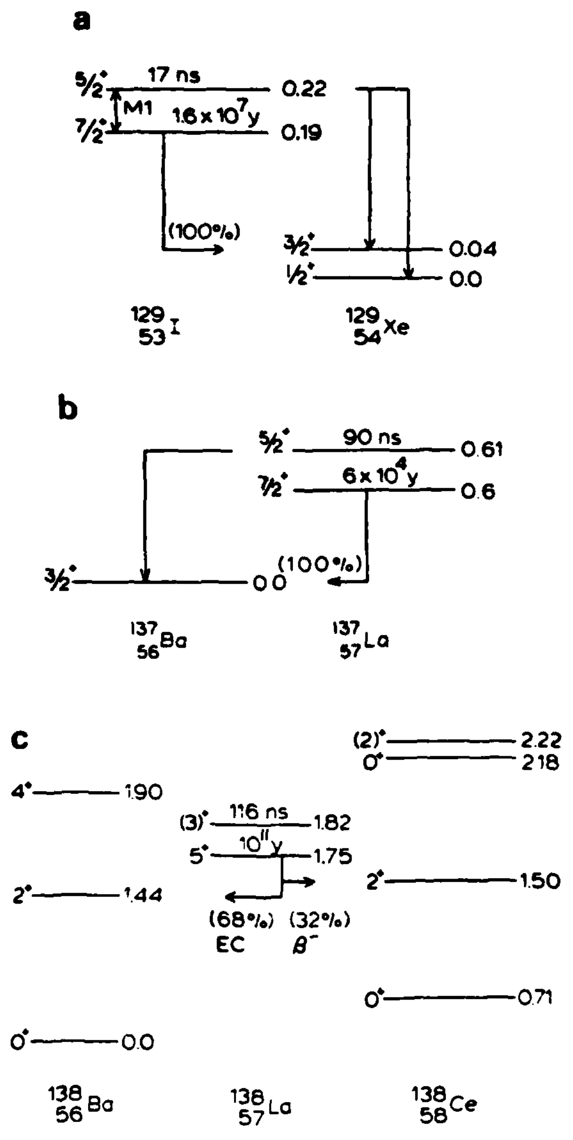

Fig. 2. (a) Level scheme for the beta-decay $^{129}_{53}I \rightarrow {}^{129}_{54}Xe$. Energy level assignments are in MeV. Data are taken from ref. [2]. (b) Same as (a) for electron capture from $^{137}_{57}La$ to $^{137}_{56}Ba$. (c) $^{138}_{57}La$ decays via a second-order forbidden decay both by β^- and by EC. Both decays are allowed from the $(3)^+$-level.

which can be of order unity or larger even for $K^2 \ll 1$.

We now want to discuss some examples for the scheme discussed thus far. $^{129}_{53}I$ decays by a highly forbidden decay to a low-lying excited state of $^{129}_{54}Xe$. From its excited $5/2^+$-state an allowed decay to the same level can take place as well as a forbidden decay to the ground state of Xe (see fig. 2a). If we take for the parameter $\Delta\omega$ the natural line width of the $5/2^+$-level and have in mind that the half life $t_{1/2}$ to be used in eq. (12) exceeds the actual half life by the internal conversion ratio, we find from eq. (13b) that we have $K^2 = 1$ for $I = 2 \times 10^{14}$ W/cm^2. A similar example relating to electronic capture is given in fig. 2b. With analogous assumptions we need in this case $I = 6 \times 10^{13}$ W/cm^2 in order to get $K^2 = 1$. Already a preliminary search of nuclear data easily yields examples with even lower ratios $\hbar\omega/mc^2$. However, this advantage tends to be more than compensated by the rapid increase in the internal conversion ratio for decreasing ω. Finally, fig. 2c exhibits an example in which a highly forbidden transition is rendered allowed by E2-absorption of one photon with $\hbar\omega = 0.072$ MeV. In this case, an intensity of 4×10^{14} W/cm^2 is required in order to achieve $K^2 = 1$. In all the preceding examples the degree of forbiddenness is reduced by two, at least. Hence a significant enhancement would already show up for $K^2 < 1$. Whereas the required intensities suggested by these examples do not seem to be extraordinary the assumed degree of monochromaticity might. Our assumption of neglecting the laser band width with respect to the natural line width of the nuclear intermediate level comes up to demanding $\Delta\omega/\omega \sim 8 \times 10^{-14}$ for $^{138}_{57}La$ (fig. 2c) and $\Delta\omega/\omega \sim 1.4 \times 10^{-12}$ for $^{129}_{53}I$ (fig. 2a). Of course, in view of eq. (13b) the requirements as to $\Delta\omega/\omega$ can be relaxed at the expense of a higher intensity.

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APPENDIX III

Volume 94A, number 3,4 PHYSICS LETTERS 7 March 1983

A NOTE ON TOTAL CROSS SECTIONS AND DECAY RATES IN THE PRESENCE OF A LASER FIELD ✩

W. BECKER, G.T. MOORE, R.R. SCHLICHER and M.O. SCULLY

Institute for Modern Optics, Department of Physics and Astronomy, University of New Mexico, Albuquerque, NM 87131, USA
and Max-Planck-Institut für Quantenoptik, D-8046 Garching near Munich, Fed. Rep. Germany

Received 10 November 1982

It is shown that in the quasiclassical regime total decay rates of neutral particles are unaffected by the application of an intense laser field and decays of otherwise stable charged particles cannot be induced. The method can also be applied to scattering problems in the presence of a laser field.

It is well known [1] that in the presence of a recoiling agent there can be a significant energy transfer between a charged particle and a laser field. This is independent of the frequency of the laser field and proportional to the square root of the intensity. For an intensity of 10^{18} W/cm^2 its maximum is of the order of 1 MeV. While such an intensity is certainly high, it is not out of reach of present day lasers [2]. Hence the idea was near at hand to investigate whether decays of elementary particles which proceed slowly or not at all due to lack of energy could be enhanced or induced by the application of an intense laser field. However, early attempts [3] (for a recent review of various decay processes in various external fields see ref. [4]) to calculate these effects brought about significant effects on decay rates only for extremely intense fields of the order of the critical field strength $E_0 = c^3m^2/e\hbar = 1.3 \times 10^{16}$ V/cm corresponding to an intensity of 4.7×10^{29} W/cm^2. The required intensity can reduce by several orders of magnitude if very small mass differences are involved [3,4], but still remains much larger than the nowadays attainable 10^{18} W/cm^2. In view of the above mentioned large energy transfer at such an intensity this non-effect has been somewhat puzzling. In this note we shall point out the reason for the fact that, while decay spectra and differential cross sections easily exhibit enormous effects due to an applied laser field, total decay rates and cross sections stubbornly ignore its presence up to an intensity of the order of the critical one. The reason will be found in the basic classicality of the interaction with the laser field as well as gauge invariance. To be specific, we shall concentrate on a plane wave laser field of arbitrary polarization and frequency decomposition so that the explicit Volkov solution [1] is available. However, our results are certainly valid for a much wider class of external fields including, e.g., constant magnetic fields. Though our approach will be completely relativistic we shall make explicit reference to nonrelativistic potential scattering in the Born approximation for the sake of simplicity. A more detailed account of the method will be given elsewhere.

The Volkov solution of the Dirac or Klein–Gordon equation in the presence of an external field, specified by the vector potential $A_\mu(x)$ with $\partial^\mu A_\mu = 0$, can be written as (we use four-vector notation with a metric such that $ab = a_0b_0 - \boldsymbol{ab}$; we let $c = 1$ but keep $\hbar$)

$$\psi_p(x) = D_p(\xi) \exp\{[-ipx + iV_p(\xi)]/\hbar\}, \tag{1}$$

✩ Work supported by the U.S. Air Force Office of Scientific Research, Contract #AFOSR-81-0128-A. Preliminary version of this work in Appl. Phys. B28 (1982) 310.

$$V_p(\xi) = \frac{e}{2pn}\int^{\xi} d\xi' \, [-2pA(\xi') + eA^2(\xi)] \,. \tag{2}$$

Here p is the momentum of the particle outside the field, $p^2 = m^2$, n is the propagation vector of the laser field, $n^2 = nA = 0$, and $\xi = nx$. The lower limit of the integral in eq. (1) is immaterial because it merely introduces a phase which cancels from all matrix elements. $D_p(x)$ is a Dirac spinor whose explicit form need not concern us here and which, of course, is absent in case of the Klein–Gordon equation.

Consider now a decay process such that an initial particle described by the wave function $\psi_p(x)$ decays into N final particles with wave functions $\psi_{p_n}(x)$ ($n = 1, ...N$). The wave functions are given by eq. (1) or are just plane waves for charged or neutral particles, respectively. The corresponding matrix element is

$$M = \int d^4x \, \bar\psi_{p_1}(x) ... \bar\psi_{p_N}(x) \Gamma \psi_p(x) \,, \tag{3}$$

where Γ specifies the coupling. The integral is conveniently performed in terms of light-like coordinates [5]

$$u = \xi/\sqrt{2} = (x^0 - x^3)/\sqrt{2} \,, \quad v = (x^0 + x^3)/\sqrt{2} \,, \tag{4}$$

where we assumed that the laser field propagates in the x^3-direction. The corresponding momenta are

$$p_u = (p^0 + p^3)/\sqrt{2} \,, \quad p_v = (p^0 - p^3)/\sqrt{2} \,, \tag{5}$$

so that

$$px = up_u + vp_v - \sum_{i=1}^{2} p_i x_i \,.$$

We then obtain

$$M = (2\pi\hbar)^3 \delta^{(3)}\Big(p - \sum_{n=1}^{N} p_n\Big)$$
$$\times \int du \exp\Big[-i\Big(p_u - \sum p_{n,u}\Big) u/\hbar\Big]$$
$$\times \exp\Big[i\Big(V_p(\xi) - \sum V_{p_n}(\xi)\Big)/\hbar\Big] \bar D_{p_1} ... \bar D_{p_N} \Gamma D_p \,, \tag{6}$$

where the three-dimensional δ-function contains the momentum components p_v and p_i ($i = 1, 2$). We shall evaluate the remaining integral by the stationary phase method. Let us abbreviate the integral by

$$I = \int_{u_0}^{\bar u_0} du \, B(u) \, e^{iA(u)/\hbar} \,. \tag{7}$$

In the quasiclassical limit $\hbar \to 0$ the stationary phase method yields

$$I \approx \sum_n \exp[iA(u_n)/\hbar] \, B(u_n) [2\pi i \hbar / A''(u_n)]^{1/2} \,, \tag{8}$$

where the sum is over all zeroes u_n of $A'(u)$ within the range $u_0 < u_n < \bar u_0$. We shall actually need $|I|^2$. In squaring eq. (8), we can discard the nondiagonal terms: since the exponent in eq. (8) is inversely proportional to $\hbar$, already small variations in the momenta and the laser field strength give rise to rapid fluctuations of the exponential. Hence due to unavoidable uncertainties in these parameters the nondiagonal terms are wiped out and we can write

$$|I|^2 = \sum_n |B(u_n)|^2 2\pi\hbar / A''(u_n)$$
$$= 2\pi\hbar \int du \, |B(u)|^2 \delta(A'(u)) \,. \tag{9}$$

In the last step we have made use of the representation

$$\delta(f(x)) = \sum \delta(x - x_n)/|f'(x_n)| \,,$$

where the sum extends over all zeroes of $f(x)$.

An obvious example where these approximations do not apply is the emission of a photon by an electron in the presence of a laser field, i.e. high-intensity Compton scattering [6]. In this case it can be shown that the quantity $A(u)$ becomes proportional to the momentum of the emitted photon. Hence $\hbar$ cancels from the exponent in eq. (7), and the condition that $A(u)/\hbar$ varies rapidly as a function of its parameters is no longer satisfied. This is related to the vanishing mass of the photon. Consequently, we have to exclude the emission of a single zero-mass particle from our considerations.

Eq. (9) shows that we can represent the decay rate $|M|^2$ as an integral over an "instantaneous" decay rate specified by a definite value of the phase u. This is a consequence of our quasiclassical approximation. This instantaneous decay rate, though for an optical laser field hardly accessible by experiment, is certainly a "truly physical quantity" in the sense of ref. [7]. As such, it must depend on the laser field only via the

kinetical momenta $p_{\mathrm{kin}}(u) = p - eA(u)$. This can be explicitly shown for eq. (6) by going over to the lightlike momentum components (5) and, if necessary, to a lightlike Dirac algebra [5]. If we now calculate the total decay rate by integrating eq. (6) over the final momenta the laser field is completely eliminated from the final state by changing the integration variables from p_n to $p_{\mathrm{kin},n}$. Obviously, this elimination relies on the representation of the decay rate as an integral over an instantaneous decay rate.

We now point out some immediate consequences which are, of course, restricted to the regime where our approximations apply: (i) the total decay rate of a neutral particle is unaffected by a laser field; (ii) if the decaying particle is charged, the total decay rate does depend on the laser field via its kinetical momentum. If we would sum over the initial momentum, too, the laser field would again be completely eliminated. This implies: (a) if the particle is stable in vacuum it remains stable in a laser field (an apparent counterexample, the decay $e \to e\gamma$, which does occur in a laser field but not in the vacuun, has been discussed above); (b) if it is unstable, the decay might be enhanced for particular momenta at the expense of being suppressed for others, so that the decay rate integrated over all momenta agrees with its vacuum value. For $|p| \gg e|A|$, however, this effect is expected to be small; (iii) for a laser field we have $A_v = (A^0 - A^3)/\sqrt{2} = 0$, hence $P_{v,\mathrm{kin}} = p_v$. Consequently, integration over $\Pi_n \, \mathrm{d}^2 p_{i,n}$ suffices to eliminate the laser field from the final state. For a neutral decay this implies, that while decay spectra plotted versus the energies p_n^0 of the decay products exhibit dramatic effects due to the laser field, spectra plotted versus $p_{n,v}$ are unaffected; (iv) the decay of a neutral particle with charged constituents is outside of the scope of our present considerations. E.g., our results do not apply to multiphotoionization of atoms, which does occur at impressive rates and energy transfers [8]. Also, whereas the enhancement of β-decay of a neutron is ruled out (conclusions to the contrary [9] originate from insufficient numerical estimates), the enhancement of nuclear β-decay is still viable, though not via a final state interaction [10,11].

It should be obvious from our derivation that it applies to a more general situation than the plane wave laser field which we considered. We expect it to apply whenever a WKB-approximation for the wave function,

$$\psi_p(x) \sim \mathrm{e}^{(-\mathrm{i}Et + \mathrm{i}S)/\hbar}\,, \tag{10}$$

is appropriate. If the classical action S depends on more than one coordinate the stationary phase approach formulated in eqs. (7) to (9) should be reconsidered. The limits of applicability of the latter are difficult to assess. It certainly breaks down when the field strength becomes comparable with the critical field strength E_0 which is a genuine quantummechanical quantity. In that event the quasiclassical approach can no longer be expected to be sufficient. However, as mentioned at the beginning, this is far beyond the realm of laboratory experiments which are within reach in the foreseeable future.

There is a close connection between our present approach and the infrared problem of quantum electrodynamics (which is classical, too). Cross sections have been shown to be essentially independent of the presence of an arbitrary number of soft photons in the initial as well as in the final state [12].

Finally, as an explicit example we shall consider nonrelativistic potential scattering in Born approximation, in order to discuss the stationary phase approach in more detail and to outline a simple way to derive the cross section. The transition amplitude (3) is in this case

$$F = -\frac{\mathrm{i}}{\hbar}\int \mathrm{d}^4x\, \psi_{p'}^*(x) V(x) \psi_p(x)\,, \tag{11}$$

where we now use the nonrelativistic limit of the Volkov solution (1), (2) in the dipole approximation,

$$\psi_p(x) = \exp\left(-\mathrm{i}px/\hbar + \frac{\mathrm{i}e}{2m\hbar}\int^t \mathrm{d}t'\,[2pA(t') - eA^2(t')]\right). \tag{12}$$

The introduction of lightlike coordinates is now obsolete, and eq. (9) yields the transition probability

$$|F|^2 = \frac{2\pi}{\hbar}|V(\hbar^{-1}(p - p'))|^2 \times \int_{-T/2}^{T/2} \mathrm{d}t\, \delta(p_0' - p_0 + (e/m)(p - p')A(t))\,, \tag{13}$$

with $p_0 = p^2/2m$, $p_0' = p'^2/2m$. Eq. (13) can be rewritten as

$$|F|^2 = (4\pi m/\hbar)|V(\hbar^{-1}(p_{\mathrm{kin}} - p'_{\mathrm{kin}}))|^2$$

$$\times \int \mathrm{d}t\, \delta(p'^2_{\mathrm{kin}} - p^2_{\mathrm{kin}}) . \quad (14)$$

This makes the exclusive dependence on the kinetical momenta obvious which has been stated above on general grounds. We evaluate eq. (13) explicitly for $A^1 = a \cos \omega t$, $A^2 = -a \sin \omega t$. The result is

$$|F|^2 = (2T/\hbar)|V(\hbar^{-1}(p - p'))|^2$$

$$\times\, [(eaq_\mathrm{T}/m)^2 - (p'_0 - p_0)^2]_+^{-1/2} , \quad (15)$$

where q_T is the transverse momentum transfer

$$q_\mathrm{T} = \left(\sum_{i=1}^{2} (p'_i - p_i)^2 \right)^{1/2} ,$$

and $x_+^\nu = x^\nu$ if $x > 0$ and zero otherwise. We note that in the limit $a = 0$ when the laser field is switched off we have

$$|F|^2 = (2\pi T/\hbar)|V(\hbar^{-1}(p - p'))|^2 \delta(p'_0 - p_0) . \quad (16)$$

The cross section is

$$\mathrm{d}\sigma = \frac{m^2}{4\pi^3 \hbar^4} \frac{|p'|}{|p|} [(eaq_\mathrm{T}/m)^2 - (p'_0 - p_0)^2]_+^{-1/2}$$

$$\times\, |V(\hbar^{-1}(p - p'))|^2 \, \mathrm{d}\Omega' \, \mathrm{d}p'_0$$

$$= \frac{1}{\pi} \frac{|p'|}{|p|} [(eaq_\mathrm{T}/m)^2 - (p'_0 - p_0)^2]_+^{-1/2}$$

$$\times \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}\Omega'} \, \mathrm{d}\Omega' \, \mathrm{d}p'_0 , \quad (17)$$

which has to be compared with the exact result [13, 14]

$$\mathrm{d}\sigma_n = \frac{|p'|}{|p|} J_n^2 (eaq_\mathrm{T}/\hbar\omega m) \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}\Omega'} \, \mathrm{d}\Omega' . \quad (18)$$

The latter is the cross section under the condition that n photons are emitted, i.e. $n\hbar\omega = p_0 - p'_0$. In view of eqs. (17) and (18) we realize that our approximations come up to

$$J_n^2(z) \approx \pi^{-1}(z^2 - n^2)_+^{-1/2} . \quad (19)$$

Eq. (19) is obtained from Debye's asymptotic formulas if (i) $n \gg 1$, (ii) the actual rapid oscillations of the Bessel functions $J_n(z)^2$ as a function of n are averaged over and (iii) a rapidly decreasing exponential tail for $n > z$ is neglected. If we are not interested in $\mathrm{d}\sigma_n$ for a particular n but only in the sum $\Sigma_{n=n_0}^{n_1} \mathrm{d}\sigma_n$ with $n_1 - n_0 \gg 1$, the assumptions (ii) and (iii) are very well satisfied. Eqs. (17) and (18) then agree if we identify $\mathrm{d}\sigma/\mathrm{d}p'_0$ with $\mathrm{d}\sigma_n/\hbar\omega$. The approximations which are inherent in eq. (19) have been extensively discussed in ref. [13]. Hence we will be content with pointing out that it is a classical approximation: both the rapid oscillations of $J_n(z)^2$ as a function of n and the exponential tail, which originates from energy transfers $n\hbar\omega$ in excess of the classical interaction energy eaq_T/m, are quantum features. As such they have dropped out of our quasiclassical approximation.

We benefitted from discussions with M. Hillary.

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APPENDIX IV

A NO-GO THEOREM CONCERNING THE ENHANCEMENT OF NUCLEAR DECAYS BY INTENSE RADIATION FIELDS

W. Becker*, R. R. Schlicher† and M. O. Scully*†

* Institute for Modern Optics
Department of Physics and Astronomy
University of New Mexico
Albuquerque, New Mexico 87131, USA

† Max-Planck-Institut für Quantenoptik
D-8046 Garching bei München
West Germany

ABSTRACT

It is shown that radiation fields well below the critical field strength cannot induce any noticeable enhancements of nuclear decay rates.

ICSU Class. No.: 03

In a previous note [1] we have shown that the total decay rate of a neutral free particle decaying into various charged particles is unaffected by the presence of an external electromagnetic plane wave field as long as the quasiclassical regime applies. This is, roughly speaking, the case if the external field is significantly weaker than the critical field strength $E_{crit} = m^2c^3/e\hbar \cong 1.3\times10^{16}$ Volts/cm, which is completely out of reach for all plane wave fields which can be generated in a laboratory.*

Recently, it has been claimed that forbidden nuclear beta-decay can be very significantly enhanced by the application of an intense radio frequency field with a field strength of $10^4 \ldots 10^5$ Volts/cm [2-4]. The idea was that due to the presence of the external field the normal selection rules should lose their relevance. The present authors contested this claim on the basis of independent calculations in the nonrelativistic limit which showed that the enhancement could be, at best, proportional to the ratio E/E_{crit} of the applied over the critical field [5]. More recent explicit calculations demonstrated that, actually, the enhancement goes with $(E/E_{crit})^2$ [6]. We also pointed out the analogy with the above mentioned no-go theorem for free particles [5]. It has been argued that this theorem since it was formulated for free neutral particles does not apply to nuclear decays [7]. This criticism is justified, in principle. It is the purpose of this note to extend the no-go theorem to the latter case, thus giving a very simple and very general argument against the possibility of altering a nuclear decay rate, be it forbidden or not, by an external electromagnetic field under the cited conditions.

The essence of our previous argument can be condensed into a few sentences: in the quasiclassical regime the differential decay rate can be written as an

* Actually, what matters here, is the field strength in the rest frame of the particle. Hence, the quasiclassical regime does not apply, for example, to ultrarelativistic electrons ($\gamma \geqslant 10^5$) in an intense focussed laser field with 10^{10} Volts/cm.

integral with respect to time over an instantaneous decay rate which depends on the instantaneous value of the vector potential $\vec{A}(t)$ of the external field. This instantaneous rate, being a gauge invariant physical quantity [8] must depend on the external field only via the kinetical momenta $\vec{p} - e\vec{A}(t)$ of the charged particles in the final state, since the decaying particle was assumed neutral. In obtaining the total decay rate, the integration over the canonical momenta $\vec{p}$ can be replaced by integration over the instantaneous kinetical momenta $\vec{p} - e\vec{A}(t)$. Thus all dependence on the external field $\vec{A}(t)$ is eliminated. If the interaction of the nuclei with the field is neglected as it is in Refs. 2 - 4, this argument carries immediately over to the case of nuclear decays, as we shall now demonstrate in detail. We shall restrict ourselves to nuclear beta-decay. However, the generalization to, for example, internal conversion or radiative transitions, is mainly notational and should be obvious.

Within the model adopted in Refs. [2 - 4] the decay is governed by the matrix element

$$M \sim \int d^4x \; e^{iE_f t} e^{-i\tilde{e}_f \vec{A}(u)\vec{r}} \phi_f^*(\vec{r})$$

$$\times \; gV \; e^{-iE_i t} e^{i\tilde{e}_i \vec{A}(u)\vec{r}} \phi_i(\vec{r})$$

$$\times \; e^{iqx/\hbar} e^{ipx/\hbar} V_p^*(u) \; , \qquad (1)$$

with

$$V_p(u) = \exp(i/\hbar)\,\frac{e}{2pn}\int^{u} du'\,(-2pA(u') + eA^2(u)) \tag{2}$$

the Volkov part of the electron wave function. We shall ignore spin for the time being. We let $c = 1$, but keep $\hbar$. In Eqs. (1) and (2), the vector potential $\vec{A}(u)$, $u = t - z$, represents the external field which propagates in the z-direction, its propagation vector being $n = (1,0,0,1)$ so that $nA = 0$. We do not have to specify its frequency decomposition nor polarization. The neutrino momentum is denoted by q, and p is the momentum of the electron outside of the field. The nuclear wave functions are $\exp(-iE_{i,f}t + i\tilde{e}_{i,f}\vec{A}(u)\vec{r})\phi_{i,f}(\vec{r})$ for the initial and final state, respectively, $\tilde{e}_{i,f}$ being effective charges [2]. The wave functions $\phi_{i,f}(\vec{r})$ are eigenfunctions of the effective nuclear Hamiltonian $H_0 = -(\hbar^2/2M_{i,f})\Delta + V(\vec{r})$ with eigenvalues $E_{i,f}$. Contrary to appearance, the nuclear wave functions do not incorporate any interaction between the nucleus and the field; the factors $\exp(i\tilde{e}_{i,f}\vec{A}\vec{r})$ are necessary in the Coulomb gauge which we have tacitly adopted by using the Volkov solution (2). We have discussed this point extensively in Ref. 5. Finally, in Eq. (1) the weak interction responsible for the beta-decay is represented by gV.

Changing the integration variables in Eq. (1) to $\vec{r}$ and u and rearranging terms we can write

$$M \sim \int du\, \rho_{fi}(\vec{q}_T + \vec{p}_T - e\vec{A}(u),\, q_z - q_0 + p_z - p_0 - E_f + E_i)$$

$$\times\, e^{(i/\hbar)(E_f - E_i + q_0 + p_0)u}\, V_p(u) \tag{3}$$

where

$$\rho_{fi}(\vec{k}_T, k_z) = \int d^3\vec{r}\ \phi_f^*(\vec{r}) gV \phi_i(\vec{r})\ e^{-i\vec{k}_T\vec{r}_T}\ e^{-ik_z z} \tag{4}$$

and we have used that $\tilde{e}_f - \tilde{e}_i = e$ [2], which is a consequence of charge conservation. The subscript T in Eqs. (3) and (4) designates two-vectors, transverse to the propagation direction of the field, e.g. $\vec{p}_T = (p_x, p_y)$. Because the integration in Eq. (4) is, due to the presence of the nuclear wave functions, restricted to $|\vec{r}| \lesssim R_0$ with R_0 the nuclear radius, the form factor (4) is a slowly varying function of its arguments. Hence we may evaluate Eq. (3) by stationary phase integration, treating ρ_{fi} as slowly varying. With obvious abbreviations we then have

$$M \sim \int du\ \rho(u)^{\frac{i}{\hbar} E(u)}$$

$$\approx \sum_n \sqrt{\frac{2\pi i\hbar}{E''(u_n)}}\ e^{(i/\hbar)E(u_n)}\ \rho(u_n)\ , \tag{5}$$

where the sum goes over all zeroes u_n of $E'(u)$. The following is strictly parallel to Ref. 1: in squaring M we can drop cross terms which would yield rapidly fluctuating contributions which average out. Consequently

$$|M|^2 \sim 2\pi\hbar \sum_n \frac{|\rho(u_n)|^2}{E''(u_n)}$$

$$= 2\pi\hbar \int du |\rho(u)|^2 \delta(E'(u))\ , \tag{6}$$

so that the differential transition rate is represented as an integral over an instantaneous rate. As argued above, the latter can only depend on $\vec{A}(u)$ via the

APPENDIX V

ALLOWED NUCLEAR BETA DECAY IN AN INTENSE LASER FIELD Δ

R. R. Schlicher*, W. Becker†, and M. O. Scully *†

†Institute for Modern Optics
University of New Mexico
Albuquerque, New Mexico 87131

*Max-Planck Institut für Quantenoptik
D-8046 Garching bei München
West Germany

INTRODUCTION

It is the purpose of this lecture to study the influence of a strong external electromagnetic field on the spectrum of the emitted particles in a nuclear beta decay and in particular on the lifetime of the radioactive nucleus. The possibility of manipulating nuclear lifetimes [1] in a laboratory is very exciting and could bring important applications. The first experimental proof of this effect is more than 30 years old [2,3]. Experimentally accessible are those nuclear decay modes which involve an interaction between the nucleus and the atomic electrons such as internal conversion [3] and orbital electron capture [4] which is closely related to beta decay. By changing the chemical environment of the atom, by applying high pressure technology, by optical excitation with strong fields, by ionization and implantation, etc. the electronic structure of the atom can be modified, which results in changes of the nuclear lifetime up to a few parts in 10^2 [4].

However, it always seemed hopeless to influence the majority of the nuclear decay modes which take place without any interaction between the decaying nucleus and its environment. All imaginable fields in a laboratory are weak compared to the strong interaction or the Coulomb field at the surface of a nucleus.

ΔSupported in part by the Air Force Office of Scientific Research

Hence, changes of these nuclear decay rates have only been investigated under conditions which are of interest in astrophysics. In particular, nuclear beta decay was studied (i) under the condition of a statistical equilibrium in stellar interiors where excited nuclear states are thermally populated [5], (ii) for single photon absorption by the emitted electron from a Planck spectrum at stellar temperatures of the order of 10^8 °K [6], and (iii) for a strong uniform constant magnetic field [7] as it exists on the surface of a pulsar.

Due to the ongoing recent progress in the development of high power lasers it is nowadays also possible to produce extraordinarily strong fields in the laboratory. For example, in the beam of a Nd-glass laser, which produces TW pulses, the intensity can be (after focusing the beam down to, say, ten wavelengths) of the order of 10^{18} W/cm^2, corresponding to a field strength of about 10^{10} V/cm. These experimental facilities suggest the theoretical treatment of nuclear decays in the presence of strong electromagnetic plane wave fields.

The effects of intense plane electromagnetic waves on different quantum processes were already investigated two decades ago [8]. The interest soon focused on the decays of elementary particle like muons and pions [9] or neutrons [10,11] under the influence of a monochromatic external field. A common feature of Refs. 7-9 and 11 is the result that the influence of the external field on the <u>total</u> decay rate depends on the ratio between the field strength and the so called critical field strength $E_c = m^2c^3/e\hbar$. This is the limit for the applicability of classical electrodynamics, beyond which quantum effects are dominant [12]. This result seems to indicate that the influence of an external field of optical frequency on the lifetime of an elementary particle is a pure quantum effect which is very small as long as E is small compared to E_c. For electrons the critical field strength is about 1.3×10^{16} V/cm and therefore still far out of the range of present laser systems.

On the other hand, Ref. 10 predicts a measurable change of the lifetime of free neutrons in presently feasible fields. Furthermore, during the last few years nuclear decays in the presence of intense plane wave fields have also been investigated. These calculations predicted appreciable enhancements of the total decay rates in the field of available lasers, both for nuclear gamma decay [13] and nuclear beta decay [14], and also recently for forbidden nuclear beta decay [15].

In these lecture notes we will follow Ref. 14 in describing the decay process. This implies the use of a modified version of the Keldysh approximation [16] which was introduced for the theoretical description of laser-ionization in the so called electric field gauge. This approximation includes two steps: (i) the interaction between the bound system and the field is

neglected, i.e. we consider the nucleus to be unaffected by the external field; (ii) in the final state the interaction between the emitted particle and the residual bound system is neglected, i.e. we neglect the Coulomb interaction between the emitted electron and the residual nucleus. The quality of the first approximation will be demonstrated in the next Section. With the second approximation we neglect a usually small effect [17]. In the framework of this approximation only the electron emitted in the nuclear beta decay couples to the external field. This is why we choose nuclear beta decay as the most promising decay mode: the smaller the mass of the emitted charged particle, the stronger its coupling to the external field and hence the stronger the laser impact on the decay process.

There are two general reasons why we might expect a change in the nuclear lifetime in the framework of this model: (i) it is well known for ordinary beta decay that taking into account the Coulomb corrections between electron and nucleus changes not only the spectrum of the electrons, but also the nuclear lifetime [17]. Hence, including the interaction of the electron with the laser field could result in the same effect; (ii) as we will show below the most likely energy transfer from, say, a Nd-laser with an intensity of $I \sim 10^{18}$ W/cm^2 to an electron emitted in a typical nuclear beta decay is of the order of MeV! Such a large energy increase leads to a much larger phase space of the emitted electrons which should result in a considerably faster decay of the nucleus.

In this lecture we shall restrict the discussion to (i) the nonrelativistic theory and (ii) to electromagnetic fields of circular polarization. This will simplify the calculations so much that they can easily be followed in detail. Strictly speaking, the nonrelativistic limit holds only if the mechanical momentum $\vec{p}-e\vec{A}$ of the electron in the field is small compared to mc. This implies a limiting condition for the kinetical electron energy E outside the field, $E = p^2/2m \ll mc^2$ and for the strength of the external field $e|A|/mc = \nu \ll 1$. This approximation seems to be unrealistic for many nuclei whose energy release in the decay is larger than the electron rest mass as well as for the fields which we intend to consider. Hence, in principle, the Dirac theory is required for the description of the electrons. Actually, relativistic effects show up in both the electron spectrum and the total decay rate. However, all the essential physical features which are introduced by the interaction with the laser field are included in the nonrelativistic theory in a very instructive way. Furthermore, as we shall demonstrate, the change of the lifetime in the presence of an external field is fairly well described by the nonrelativistic theory for all real nuclei and arbitrary field strengths, not only in the nonrelativistic limit $|\vec{p}-e\vec{A}| \ll mc$. Hence, we will present here only the nonrelativistic theory. However, all the numerical results shown are gained from the relativistic beta decay theory for allowed

transitions with Dirac wave functions and V-A interaction. This theory will be published elsewhere.

We will first derive the wave functions of the different particles involved in the nuclear beta decay. This is based on Ref. 18. Whhat follows will be completely independent of Ref. 18. Next the partial transition rates will be calculated in the electric field gauge and in the radiation gauge. We will then evaluate the electron distributions and finally we will discuss the nuclear lifetime.

WAVEFUNCTIONS

For the calculation of the wavefunctions of the different particles involved in a nuclear beta decay we have to recall some points discussed in Ref. 18. We can obtain the Hamiltonian of a particle with charge e interacting with an external electromagnetic field from the Hamiltonian $H = p^2/2m + V(\vec{r})$ of a particle in a potential V with the help of the substitution

$$\vec{p} \rightarrow \vec{p} - e\vec{A}^g(\vec{r},t)\ , \quad H \rightarrow H^g - e\,U^g(\vec{r},t)\ .$$

This procedure yields the Hamiltonian

$$H^g = \frac{1}{2m}\left(\vec{p} - eA^g(\vec{r},t)\right)^2 + e\,U^g(\vec{r},t) + V(\vec{r}) . \tag{2.1}$$

The vector potential $\vec{A}^g(\vec{r},t)$ and the scalar potential $U^g(\vec{r},t)$ are related to the electric field $\vec{E}(\vec{r},t)$ and the magnetic field $\vec{B}(\vec{r},t)$ by

$$\vec{E}(\vec{r},t) = -\vec{\nabla}U^g(\vec{r},t) - \frac{\partial}{\partial t}\vec{A}^g(\vec{r},t)\ ,$$
$$\vec{B}(\vec{r},t) = \vec{\nabla}\times\vec{A}^g(\vec{r},t)\ . \tag{2.2}$$

The index g denotes the gauge freedom in the potentials. $\vec{E}$ and $\vec{B}$ remain unchanged if we transform from a gauge $(\vec{A}^g, U^g)$ to a gauge $(\vec{A}^{g'}, U^{g'})$ according to

$$\vec{A}^g(\vec{r},t) \rightarrow \vec{A}^{g'}(\vec{r},t) = \vec{A}^g(\vec{r},t) + \vec{\nabla}\chi(\vec{r},t)\ ,$$
$$U^g(\vec{r},t) \rightarrow U^{g'}(\vec{r},t) = U^g(\vec{r},t) - \frac{\partial}{\partial t}\chi(\vec{r},t)\ . \tag{2.3}$$

where χ is an arbitrary function of $\vec{r}$ and t .

Also, the state vector $|\Psi^g\rangle$, which is a solution of the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\Psi^g\rangle = H^g |\Psi^g\rangle \quad , \tag{2.4}$$

depends on the gauge. The wave function in a gauge g is transformed to another gauge g' by the unitary transformation

$$\Psi^g(\vec{r},t) \to \Psi^{g'}(\vec{r},t) = e^{ie\chi(\vec{r},t)/\hbar}\, \Psi^g(\vec{r},t). \tag{2.5}$$

The Hamiltonian has to be distinguished from the energy operator

$$\varepsilon^g = \frac{1}{2m}\left(\vec{p} - e\vec{A}(\vec{r},t)^2\right). \tag{2.6}$$

This (unperturbed) energy operator is a physical quantity in the sense of Ref. 19, since it transforms under a gauge transformation like

$$\varepsilon^{g'} = e^{ie\chi/\hbar}\, \varepsilon^g\, e^{-ie\chi/\hbar}.$$

In contrast, the Hamiltonian is an unphysical quantity since it transforms like

$$H^{g'} = e^{ie\chi/\hbar} H^g e^{-ie\chi/\hbar} - e\frac{\partial}{\partial t}\chi .$$

This distinction is important for the definition of an unperturbed state as discussed in Ref. 18.

Throughout these lecture notes we will describe the laser field by a uniform electric field $\vec{E}(t)$, neglecting the magnetic field. This long wavelength or dipole approximation is well justified for nuclear beta decay since the wavelength of visible light is about $5\text{x}10^{-5}$ cm, whereas the radius of a nucleus is of the order of $5\text{x}10^{-13}$ cm. Hence, when calculating matrix elements of the nucleus-laser interaction the external field can be considered as constant over the integration area. Within this long wavelength approximation two gauges are most frequently used: the electric field gauge (E-gauge) with

$$\vec{A}^E(t) = 0 \;, \quad \mathcal{U}^E(\vec{r},t) = -e\vec{r}\,\vec{E}(t) \tag{2.7}$$

and the radiation gauge (R-gauge) with

$$\vec{A}^R(t) = -\int_{t_0}^{t} d\tau\, \vec{E}(\tau)\,, \quad \mathcal{U}^R(\vec{r},t) = 0\,. \tag{2.8}$$

Both gauges are related by the gauge transformation

$$\chi_{R\to E}(\vec{r},t) = -\vec{A}^R(\vec{r},t)\vec{r} = \vec{r}\int_{t_0}^{t} d\tau\, \vec{E}(\tau)\,. \tag{2.9}$$

For the nuclear wave functions to be used for the calculation of the beta-decay we shall adopt a simple one-particle shell model description: the nucleus is divided into a "valence" nucleon and an inert core which is affected neither by the field nor by the decay but generates the potential $V(\vec{r})$ for the valence nucleon which decays. The valence nucleon has an effective charge e_N and mass M. Both do not have to be specified; we have just to recall that because of charge conservation the effective charges of the initial and final valence nucleon will satisfy the relation $e_{N,i} - e_{N,f} = \pm e$ for beta$^{\mp}$-decay ($e = -|e|$ denotes the electron charge).

It is convenient to use the E-gauge. The nuclear wave-function $\Psi_N(r,t)$ is then a solution of the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi_N(\vec{r},t) = \left(\frac{\vec{p}^{\,2}}{2m} + V(\vec{r}) - e_N\vec{E}(t)\vec{r}\right)\Psi_N(\vec{r},t)\,. \tag{2.10}$$

The interaction term $-e_N\vec{E}\vec{r}$ is at the nuclear surface of the order of $10^{-2} - 10^{-1}$eV if we apply a field of 10^{10} V/cm. This is completely negligible compared to the nuclear binding potential or to the Coulomb potential at the nuclear surface, which are of the order of MeV. A noticeable modification of a bound nuclear state would generally require field strengths close to the critical one. We shall therefore assume that the initial and final nuclear states do not interact with the external field. This is the first part of the Keldysh approximation.

As discussed in Ref. 18, the noninteracting state is defined as an eigenstate of the unperturbed energy operator (2.6), not as a solution of the Schrodinger Eq. (2.4) in which the potentials $\vec{A}^g$ and U^g are set equal to zero. In the E-gauge this distinction does not play a role since the Hamiltonian H^E and the energy $\mathcal{E}^E$ coincide. The noninteracting nuclear state is then a solution of Eq. (2.10) in which we drop the nucleus-field interaction term $-e_N\vec{E}\vec{r}$:

$$\Psi_N^E(\vec{r},t) = e^{-iE_N t/\hbar}\,\Phi_N(\vec{r}), \tag{2.11}$$

where Φ_N is an energy eigenstate

$$\left(\frac{\vec{P}^2}{2M} + V(\vec{r})\right)\Phi_N(\vec{r}) = E_N\,\Phi_N(\vec{r}). \tag{2.12}$$

If we want to express the noninteracting nuclear state in the R-gauge we have to be cautious. In order to obtain an eigenstate of the energy operator ε^R (2.6) with the same constant energy eigenvalue E_N we must apply the unitary transformation (2.5) and (2.9). We then obtain a wavefunction which is different from Eq. (2.11)

$$\Psi_N^R(\vec{r},t) = e^{ie_N\vec{A}^R(t)\vec{r}/\hbar}\,e^{-iE_N t/\hbar}\,\Phi_N(\vec{r}). \tag{2.13}$$

From now on we shall drop the superscript in $\vec{A}^R$ and refer with the notation $\vec{A}$ to the vector potential in the R-gauge (2.8).

For the calculation of the electron wavefunction we use the second part of the Keldysh approximation and neglect the Coulomb interaction of the emitted electron with the residual nucleus. However, the interaction between the electron and the external field should be taken into account exactly. Hence, we need an exact solution of the Schrödinger equation for a free particle ($V = 0$) in the external field $\vec{A}(t)$.

We notice now that it is more convenient to solve this problem in the R-gauge

$$i\hbar\frac{\partial}{\partial t}\Psi_e^R(\vec{r},t) = \frac{1}{2m}\left(\vec{p} - e\vec{A}(t)\right)^2\Psi_e^R(\vec{r},t) \tag{2.14}$$

instead of the E-gauge

$$i\hbar\frac{\partial}{\partial t}\Psi_e^E(\vec{r},t) = \left(\frac{\vec{P}^2}{2m} - e\vec{r}\,\vec{E}(t)\right)\Psi_e^E(\vec{r},t) \tag{2.15}$$

since the canonical momentum $\vec{p}$ of the free particle is a constant of motion in the R-gauge, but not in the E-gauge. It is well known in classical mechanics [20] that the i-th component p_i of the canonical momentum $\vec{p}$ is conserved if neither the vector potential $\vec{A}^g$ nor the scalar potential U^g depend on the i-th component x_i of the space coordinate r. In particular, if $\vec{A}^g$ and U^g

are spatially uniform $\vec{p}$ is conserved. This also holds true in quantum mechanics: an operator is a constant of motion if it does not explicitly depend on time and if it commutes with the Hamiltonian H^g. Hence, if $\vec{A}^g$ and U^g do not depend on $\vec{r}$ the eigenvalue of the canonical momentum p is a constant of motion. This is true for the Hamiltonian H^R in the R-gauge (2.8) with the long wavelength approximation for the field. If we make the additional assumption that the vector potential $\vec{A}(t)$ is switched on and off initially and finally, i.e. $\vec{A}(t) = 0$ for $|t| > t_0$, we can identify the conserved canonical momentum with the momentum outside of the field, which would be measured by a spectrometer. Again, this holds only true in the R-gauge. On the other hand the canonical momentum is not a physical quantity in the sense of Ref. 19. Hence its eigenvalues are different in different gauges. Especially in the E-gauge (2.7), where the vector potential vanishes and the scalar potential does depend on $\vec{r}$, the canonical momentum $\vec{p}$ coincides with the mechanical momentum $\vec{\pi}(t) = \vec{p} - e\vec{A}\,(t)$ and its eigenvalue is no longer conserved.

Since it is always convenient to exploit the existence of conserved quantities, we like to solve Eq. (2.14) instead of Eq. (2.15). Only in the R-gauge can we make the following ansatz for the electron wavefunction: Ψ_e^R is characterized by the eigenvalue $\vec{p}$ of the canonical momentum and we assume that it factorizes into an unperturbed plane wave and a function f(t) which depends only on time, since also the field depends in the long wave length approximation only on time

$$\Psi_e^R(\vec{r},t) = f(t)\, e^{-i(Et - \vec{p}\vec{r})/\hbar} .$$

In the nonrelativistic theory E is related to the canonical momentum by

$$E = \frac{\vec{p}^{\,2}}{2m} . \tag{2.16}$$

Only in the absence of the field denotes E the kinetic energy. By inserting the ansatz into the Schrödinger equation (2.14) and using Eq. (2.16) one obtains the equation of motion of f(t)

$$i\hbar \frac{\partial}{\partial t} f(t) = \left(-\frac{e}{m}\vec{p}\vec{A}(t) + \frac{e^2}{2m}\vec{A}^{\,2}(t) \right) f(t) .$$

Integrating this equation yields the exact solution of Eq. (2.14):

$$\Psi_e^R(\vec{r},t) = V^{-1/2} \times$$

$$\times \exp\left\{-\frac{i}{\hbar}\left[Et - \vec{p}\vec{r} - \frac{1}{2m}\int_{t_0}^{t} d\tau\left(2e\vec{p}\vec{A}(\tau) - e^2\vec{A}^2(\tau)\right)\right]\right\} \tag{2.17}$$

where V denotes the normalization volume. The lower limit of integration in Eq. (2.17) only contributes a constant phase to the wavefunction and is therefore insignificant. The analogous solution of the Dirac equation is known as the Volkov solution [21].

It is easy to check that the wavefunction (2.17) is an eigenstate of the operator of the canonical momentum with eigenvalue $\vec{p}$. Since the operators of the canonical momentum $\vec{p}$ and of the mechanical momentum $\vec{\pi}^R = \vec{p} - e\vec{A}$ commute in the long wavelength approximation, Ψ_e^R is also an eigenstate of $\vec{\pi}^R$ and of the operator of the kinetic energy in the R-gauge

$$\varepsilon^R \Psi_e^R(\vec{r},t) = \frac{1}{2m}\left(\vec{p} - e\vec{A}(t)\right)^2 \Psi_e^R(\vec{r},t). \tag{2.18}$$

The nonrelativistic Volkov solution Ψ_e^R has a time dependent energy eigenvalue, in contrast to the non-interacting state Ψ_N^R (2.13) which has a constant energy eigenvalue E_N.

In order to obtain the solution of Eq. (2.15) in the E-gauge we have to carry out the gauge transformation (2.5) and (2.9) on Ψ_e^R with the result

$$\Psi_e^E(\vec{r},t) = e^{-ie\vec{A}(t)\vec{r}/\hbar}\,\Psi_e^R(\vec{r},t)$$

$$= V^{-1/2}\exp\left\{-\frac{i}{\hbar}\left[\left(\vec{p} - e\vec{A}(t)\right)\vec{r} - \frac{1}{2m}\int_{t_0}^{t} d\tau\left(\vec{p} - eA(\tau)\right)^2\right]\right\} \tag{2.19}$$

where we used Eq. (2.16). We note that Ψ_e^E depends only on the eigenvalue of the mechanical momentum $\vec{\pi}(t) = \vec{p} - e\vec{A}(t)$. This is a physical quantity and has therefore the same value in any gauge. Here we expressed $\vec{\pi}$ by the gauge dependent canonical momentum and vector potential, both given in the R-gauge.

Throughout these lecture notes we shall describe the laser field by a circularly polarized monochromatic plane wave with frequency ω, propagating in $\hat{e}_3$-direction

$$\vec{E}(t) = E_0 \left(\hat{e}_1 \sin\omega t + \hat{e}_2 \cos\omega t \right). \quad (2.20)$$

The corresponding vector potential in the R-gauge reads

$$\vec{A}(t) = A_0 \left(\hat{e}_1 \cos\omega t - \hat{e}_2 \sin\omega t \right), \; A_0 = \frac{E_0}{\omega}. \quad (2.20b)$$

By inserting this vector potential into Ψ_e^R (2.17) and dropping the phase which results from the lower integration limit we obtain

$$\Psi_e^R(\vec{r},t) = V^{-1/2} \exp\left[-\frac{i}{\hbar}\left(E + \frac{\nu^2}{2} mc^2\right)t - \vec{p}\vec{r}\right] \times \quad (2.21)$$

$$\times \exp\left[iz\sin(\omega t + \varphi)\right].$$

Here we introduced the azimuthal and polar angles ϕ and θ of the momentum $\vec{p}$

$$p_1 = p_T \cos\varphi, \; p_2 = p_T \sin\varphi, \; p_T = \sqrt{p_1^2 + p_2^2} = p\sin\vartheta. \quad (2.22)$$

The field dependence is expressed by the dimensionless parameter ν, which was introduced earlier

$$\nu = \frac{eA_0}{mc} = \frac{eE_0}{m\omega c} = \frac{mc^2}{\hbar\omega}\frac{E_0}{E_c}. \quad (2.23)$$

E_c is again the critical field strength $E_c = m^2c^3/e\hbar = 1.3\times10^{16}$ V/cm. For practical calculations it can be convenient to express ν either by the intensity I and wavelength λ of the field

$$\nu^2 = \frac{e^2 \lambda I}{2\pi^2 \varepsilon_0 m^2 c^5} = 7.3\times10^{-19} \lambda^2 [\mu m^2]\, I[W/cm^2],$$

or by the photon density ρ and the wavelength λ

$$\nu^2 = \frac{e^2 \hbar \lambda \rho}{\pi \varepsilon_0 m^2 c^3} = 4.3\times10^{-27} \lambda [\mu m]\, \rho [cm^{-3}].$$

Finally, the dimensionless amplitude z of the oscillating term in the exponent of Eq. (2.21) is given by

$$z = \frac{\nu c p_T}{\hbar\omega} = \frac{\nu c p}{\hbar\omega} \sin\vartheta \,. \tag{2.24}$$

For circular polarization the $\vec{A}^2$-term in Eq. (2.17) is constant, $\vec{A}^2(t) = A_0^2$, and gives rise to the field dependent energy shift $\nu^2 mc^2/2$ in Ψ_e^R (2.21). In a relativistic context, this term is usually interpreted as a contribution to an effective mass. This becomes apparent from the relativistic Hamiltonian

$$H = \left(m^2c^4 + c(\vec{p} - e\vec{A})^2\right)^{1/2}$$
$$= \left((m^2 + e^2A_0^2/c^2)c^4 + c^2\vec{p}^{\,2} - 2e\vec{p}\vec{A}\right)^{1/2}.$$

Hence, in the following, we shall refer to this term as the effective mass correction. It will play a crucial role in the following Sections.

The $\vec{p}\,\vec{A}$ term of Eq. (2.17) is the origin of the oscillating exponent exp(iz sin) in Ψ_e^R (2.21) and responsible for the fact that the kinetic energy (2.18) is not a constant of motion. It can be interpreted in terms of the electron undergoing field induced multiphoton transitions. We see this with the help of the generating function of the Bessel function $J_n(z)$

$$e^{iz\sin\alpha} = \sum_{n=-\infty}^{\infty} J_n(z)\, e^{in\alpha} \,. \tag{2.25}$$

This enables us to rewrite the nonrelativistic Volkov wavefunction in the form

$$\Psi_e^R(\vec{r},t) = V^{-1/2} \sum_{n=-\infty}^{\infty} J_n(z)\, e^{in\varphi} \times$$
$$\times \exp\left\{-\frac{i}{\hbar}\left[\left(E + \frac{\nu^2}{2}mc^2 - n\hbar\omega\right)t - \vec{p}\vec{r}\right]\right\}. \tag{2.26}$$

Although we only deal with classical fields we can interpret the wavefunction (2.26) in terms of an infinite sum over n absorbed or emitted photons.

The time dependent part of Ψ_e^R includes an effective energy which changes in discrete steps of $\hbar\omega$. This represents multiphoton processes of any order n. We can estimate the most likely

energy transfer from the field to the electron by considering the behavior of $J_n^2(z)$ as a function of the order n for fixed z. When the rapid oscillations are averaged over, $J_n^2(z)$ increases slowly with increasing $|n|$ until it reaches its maximum around $|n| = z$. For $|n| > z, J_n(z)$ as a function of n decreases rapidly to zero. Hence it is most likely that about z photons are absorbed by the electron, corresponding to an energy transfer $z\hbar\omega = \nu c p_T$. In an external field with ν of the order of unity the electron can therefore absorb energy from the field up to the order of its own kinetic energy, typically some MeV for nuclear beta decay. This strong energy absorption encourages us to expect a considerable laser effect on nuclear beta decay. However, we realize already that this will be counteracted by the increase of the electron's effective mass.

Finally, we need the neutrino wavefunction. Since the neutrino neither interacts with the laser field nor with the residual nucleus, it is well described by a plane wave solution of the Klein-Gordon equation

$$\Psi_\nu(\vec{r},t) = V^{-1/2} \exp\left[-\frac{i}{\hbar}(E_\nu t - \vec{q}\vec{r})\right] \tag{2.27}$$

with the relativistic dispersion

$$E_\nu = c|\vec{q}|. \tag{2.28}$$

TRANSITION RATE

We now intend to calculate in the framework of the model derived in the last Section the effects of a strong laser field on measurable quantities like the spectrum of the emitted electrons and the lifetime of the nucleus. The transition amplitude reads in first order of perturbation theory with respect to the weak interaction

$$F(T) = -\frac{i}{\hbar} \int_{-T/2}^{T/2} dt \langle \Psi_e \Psi_\nu \Psi_{N,f} | gV | \Psi_{N,i} \rangle. \tag{3.1}$$

The weak interaction is denoted by (gV) where V is a dimensionless operator and g is the coupling constant for beta decay, $g = 1.4\times10^{-49}$ erg cm^3. According to the Keldysh approximation the electron wavefunction Ψ_e is given by the exact solution (2.21) or (2.18) for a free particle in a circularly polarized field and the nuclear wavefunctions $\Psi_{N,i}$ and $\Psi_{N,f}$ are given by the noninteracting solution (2.11) or (2.13), respectively. The

uncharged neutrino is described by the plane wave (2.27). By taking the wavefunctions of all charged particles consistently in the same gauge, either the E or the R-gauge, we obtain in any case

$$F(T) = -\frac{i}{\hbar V} \int_{-T/2}^{T/2} dt \; e^{\frac{i}{\hbar}\left(E + \frac{\nu^2}{2} mc^2 + E_\nu - Q\right)t} \times$$

$$\times \, e^{-iz\sin(\omega t + \varphi)} \langle e^{\frac{i}{\hbar}[\vec{p} - e\vec{A}(t) + \vec{q}]\vec{r}} \Phi_f \, | \, gV \, | \, \Phi_i \rangle . \tag{3.2}$$

If we choose the wavefunctions in the R-gauge we have to make use of the relation $e_{N,f} - e_{N,i} = -e$ in order to obtain Eq. (3.2). For the energy balance one has to take into consideration the mass of the electron which is created during the beta decay. Hence the value Q in Eq. (3.2) is given by

$$Q = E_{N,i} - E_{N,f} - mc^2 . \tag{3.3}$$

Since we are only interested in allowed nuclear beta decay we can drop the factor $\exp\{i(\vec{p}-e\vec{A}+\vec{q})\vec{r}/\hbar\}$ in the matrix element. The physical justification for this approximation is the fact the electron and the neutrino do not carry away any angular momentum in an allowed beta decay so that the spatial dependence of their wavefunctions can be neglected. In the nonrelativistic theory this is a fair approximation. The nonrelativistic limit implies $|\vec{p} - e\vec{A}(t)| \ll mc$, as discussed above. Since the spatial integraion in the matrix element extends over the radius R of the nucleus, which is of the order of some 10^{-13}cm, the exponential factor can be estimated for all nuclei by

$$|(\vec{p} - e\vec{A})\vec{r}/\hbar| < |\vec{p} - e\vec{A}|R/\hbar \ll Rmc^2/\hbar c \leq 0.02 \ll 1 .$$

Since we now dropped the factor $\exp\, ie\vec{A}\vec{r}/\hbar$, the discussion of the preceding Section on the question of when to take which wave function, viz. (2.11) or (2.13), appears obsolete. Nevertheless, we have emphasized this point because it is important in principle and of crucial significance in the case of forbidden beta-decay [16, 32].

By the same arguments the space dependent factor $\exp(i\vec{q}\vec{r}/\hbar)$ in the neutrino wavefunction can be replaced by unity. If the nuclear matrix element is abbreviated by

$$g V_{fi} = \langle \Phi_f \mid g V \mid \Phi_i \rangle ,$$

the transition amplitude for allowed beta decay takes the form

$$F(T) = -\frac{i}{\hbar V}\, g V_{fi} \int_{-T/2}^{T/2} dt\, \exp\left(-iz\sin(\omega t+\varphi)\right) \times \tag{3.4}$$
$$\times \exp\left[\frac{i}{\hbar}\left(E+\frac{\nu^2}{2}mc^2+E_\nu-Q\right)t\right].$$

With the help of the expansion (2.25) Eq. (3.4) can be rewritten in the form

$$F(T) = -\frac{i}{\hbar V}\, g V_{fi} \sum_{n=-\infty}^{\infty} J_n(z)\, e^{-in\varphi} \times$$
$$\times \int_{-T/2}^{T/2} dt\, \exp\left[\frac{i}{\hbar}\left(E+\frac{\nu^2}{2}mc^2+E_\nu-Q-n\hbar\omega\right)t\right]. \tag{3.5}$$

This representation again allows for a simple interpretation in terms of n photons emitted or absorbed by the electron.

For the calculation of the decay rate we only need the transition amplitude (3.5) in the limit $T \to \infty$:

$$\lim_{T\to\infty} F(T) = -\frac{i}{\hbar V}\, g V_{fi} \sum_{n=-\infty}^{\infty} J_n(z)\, e^{-in\varphi} \times$$
$$\times\, 2\pi\, \delta\left(\left(E+\frac{\nu^2}{2}mc^2+E_\nu-Q-n\hbar\omega\right)/\hbar\right). \tag{3.6}$$

With the help of the standard relation [22]

$$\left(2\pi\,\delta(E)\right)^2 \to 2\pi T\, \delta(E)$$

we obtain the transition rate per unit time

$$w = \lim_{T\to\infty} \frac{1}{T} |F(T)|^2$$

$$= \frac{2\pi}{\hbar} \frac{1}{V^2} |gV_{fi}|^2 \sum_{n=-\infty}^{\infty} J_n^2(z)\, \delta\left(E + \frac{\nu^2}{2} mc^2 + E_\nu - Q - n\hbar\omega\right). \tag{3.7}$$

Only diagonal terms $n = n'$ contribute to $|F(T)|^2$ in the limit $T \to \infty$ since the off-diagonal terms yield products of two delta functions with different arguments. Due to reasons which will become clear below it is convenient to introduce the dimensionless quantity

$$n_0 = \left(E + \frac{\nu^2}{2} mc^2 - Q\right)/(\hbar\omega).$$

Eq. (3.7) can then be rewritten in the form

$$W = \frac{2\pi}{\hbar^2 V^2 \omega} |gV_{fi}|^2 \sum_{n=-\infty}^{\infty} J_n^2(z)\, \delta\left(n - n_0 - \frac{E_\nu}{\hbar\omega}\right). \tag{3.8}$$

In the next step we will simplify the sum over the Bessel functions. In high power laser fields the argument z of the Bessel function can achieve values of 10^3 to 10^6 as discussed above. Hence, up to 10^6 terms can contribute to the sum (3.8). An excellent approximation for Bessel functions of large order n and for $z > |n|$ is Debye's asymptotic expansion [23]:

$$J_n(z) = \sqrt{\frac{2}{\pi}}\,(z^2 - n^2)^{-1/4} \cos\left(\sqrt{z^2 - n^2} - n \arccos\frac{n}{z} - \frac{\pi}{4}\right).$$

The rapidly decreasing exponential tail of $J_n(z)$ for $|n| > z$ will be neglected, i.e. we set $J_n(z) = 0$ for $|n| > z$. This approximation is only exact in the limit $z \to \infty$. According to Eq. (2.24) this corresponds to the classical limit $\hbar \to 0$. As long as z is a large number this classical limit is well justified. Furthermore, after squaring $J_n(z)$ we can average over the oscillations in $J_n^2(z)$ since it is hardly possible in any experiment to determine the intensity parameter ν and therefore also z to such an accuracy that the phase of the Bessel functions is precisely defined. Also these rapid oscillations of $J_n^2(z)$ as a function of n are a quantum feature [24]. This procedure then yields a powerful approximation for the square of Bessel functions

$$\overline{J_n^2}(z) \cong \frac{1}{\pi} \frac{\theta(z^2 - n^2)}{\sqrt{z^2 - n^2}} . \qquad (3.9)$$

The θ-function denotes the usual step function $\theta(x) = 1$ for $x > 0$ and $= 0$ for $x < 0$.

Since we are not interested in a particular term of Eq. (3.8) with a fixed number n of transferred photons, but only in the total effect of very many multiphoton terms, we can replace the sum in Eq. (3.8) by an integral. By inserting the approximation (3.9) into Eq. (3.8) and carrying out the n-integration we obtain the transition rate per unit time in the quasiclassical limit

$$w = \frac{2\,|gV_{fi}|^2}{\hbar^2 V^2 \omega} \frac{\theta\left(z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2\right)}{\sqrt{z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2}} . \qquad (3.10)$$

If we recall that z and n are proportional to $(\hbar\omega)^{-1}$ we notice that in the quasiclassical approximation, which applies in the limit $\hbar\omega \ll mc^2$, the transition rate w becomes independent of the field frequency ω. It depends on the external field only via the intensity parameter ν (2.23).

ALTERNATIVE DERIVATIONS OF THE QUASICLASSICAL TRANSITION RATE

There are two further instructive and easy methods to derive Eq. (3.10) which illustrate its classical character. In the first part of the current Section we calculate the time integral for the transition amplitude (3.4) with the help of the stationary phase method [25] instead of expanding the integrand in terms of Bessel functions. The method of stationary phase approximates the integral over a rapidly oscillating function by the contributions from the regions around stationary points to the integrand [26]:

$$\int_a^b dt\, e^{ixf(t)} \cong \sum_n \sqrt{\frac{2\pi i}{x f''(t_n)}}\, e^{ixf(t_n)} .$$

Here x is a large positive variable and f(t) a real function of the real variable t, so that the integrand is rapidly oscillating unless f(t) is stationary. Hence the major contribution to the

value of the integral arises from the vicinity of the points t_n at which $f'(t_n) = 0$. The sum over n extends over all stationary points in the interval [a,b]. The stationary phase method becomes exact in the limit $x \to \infty$, i.e. for infinitely large exponents. Hence the use of this approximation method for the time integration in Eq. (3.4) implies the transition to the classical limit $\hbar \to 0$ (i.e. $z \to \infty$) already in the transition amplitude F(t).

In order to apply the stationary phase method to Eq. (3.4) we first have to determine the stationary points t_n of the exponent of the integrand:

$$\cos(\omega t_n + \varphi) = \frac{1}{z}\left(n_0 + \frac{E_\nu}{\hbar\omega}\right).$$

In order to obtain stationary points we find again the condition $z^2 \geqslant (n_0 + E_\nu/\hbar\omega)^2$. We furthermore note that the second derivative of the exponent has the same form

$$\pm\,\omega^2\sqrt{z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2}$$

at all stationary points. Thus we obtain for the transition amplitude in the semiclassical limit

$$F(T) = -\frac{\sqrt{-2\pi i}}{\sqrt{\hbar\omega}}\, g V_{fi}\, \frac{\theta\left(z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2\right)}{\left(z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2\right)^{1/4}} \times$$

$$\times \sum_{[-T/2,\, T/2]} \exp\left\{ i\left[\left(n_0 + \frac{E_\nu}{\hbar\omega}\right)\left(\arccos\frac{n_0 + \frac{E_\nu}{\hbar\omega}}{z} - \varphi\right) \mp \right.\right. \tag{4.1}$$

$$\left.\left. \mp\sqrt{z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2} - 2\pi n\left(n_0 + \frac{E_\nu}{\hbar\omega}\right)\right]\right\}.$$

Cross terms from different stationary points will once again not contribute to the transition rate $|F(T)|^2$ since the phases at the stationary points t_n are randomly distributed for all practical purposes so that the cross terms cancel each other in the limit $T \to \infty$. We have two stationary points per time interval $2\pi/\omega$ and hence $\omega T/\pi$ stationary points in the entire integration region $[-T/2, T/2]$. We thus find from counting all stationary points in $|F(T)|^2$ the transition rate per unit time

$$W = \lim_{T\to\infty} \frac{1}{T} \, | F(T) |^2$$

$$= \frac{2 \, |g \, V_{fi}|^2}{V^2 \hbar^2 \omega} \; \frac{\theta\left(z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega} \right)^2 \right)}{\sqrt{z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega} \right)^2}} \, . \qquad (4.2)$$

This result is identical with the quasiclassical approximation (3.10) of the Bessel function approach.

There is an even simpler argument leading to Eq.(4.2) which is classical from the outset. We can write the transition rate per unit time for the decay in the absence of the laser field as

$$W_{free} = \frac{2\pi}{\hbar V^2} \, |g V_{fi}|^2 \, P_{free}(E) \, , \qquad (4.3)$$

where

$$P_{free}(E) = \delta (Q - E_\nu - E) . \qquad (4.4)$$

is the electron energy distribution function and $E = p^2/2m$ the kinetic energy. Of course, in the absence of the field the electron energy E is, for specified Q, a function of the neutrino energy E_ν only. In the presence of the field the kinetic energy of an electron is no longer conserved but time-dependent (cp. Eq. (2.61)), so that

$$E(t) = \frac{1}{2m} \left(\vec{p} - e \vec{A}(t) \right)^2$$

$$= \frac{\vec{p}^{\,2}}{2m} + \frac{\nu^2}{2} m c^2 - \hbar \omega z \cos(\omega t + \varphi) \qquad (4.5)$$

with $\vec{p}$ the conserved canonical momentum. We now assume that when the field is switched on, V_{fi} in Eq. (4.3) is unaffected but the electron energy distribution $P_{free}(E)$ is changed into

$$P(E) = \frac{\omega}{2\pi} \int_{t_0}^{t_0 + 2\pi/\omega} dt \; \delta \left(Q - E_\nu - E(t) \right) . \qquad (4.6)$$

This is the time average over the instantaneous electron energy distribution. If we recall that

$$\delta(f(x)) = \sum_i \frac{\delta(x-x_i)}{|f'(x_i)|} ,$$

where the sum is over all zeroes x_i of $f(x)$, and notice that $Q-E_\nu-E(t)$ has two zeroes within $t_0 \leq t \leq t_0 + 2\pi/\omega$, we find that

$$w = \frac{2\pi}{\hbar V^2} |g V_{fi}|^2 P(E) \tag{4.7}$$

agrees with Eq. (4.2). This procedure illustrates once again that the approximation (3.9) for the Bessel functions as well as the stationary phase method correspond to a purely classical treatment of the electron-laser interaction.

By comparing Eqs. (4.4) and (4.6) we can already anticipate a general feature of the electron energy distribution which will be more extensively discussed in the next Section. Due to the field, the kinetic energy

$$E_{free} = \frac{\vec{p}_{free}^{\,2}}{2m} = Q - E_\nu$$

in Eq. (4.4) is replaced by E(t) of Eq. (4.5). Now, if $|e\vec{A}| >> |\vec{p}_{free}|$, $|\vec{p}|$ will be much larger than $|\vec{p}_{free}|$ in order that

$$\vec{p}_{free}^{\,2} = (\vec{p} - e\vec{A}(t))^2$$

can be satisfied as prescribed by Eqs. (4.4) and (4.6). Hence, for strong fields, we expect the electron energy spectrum to extend to much higher energies.

ELECTRON DISTRIBUTIONS

In order to obtain the angular distribution and the energy spectrum of the emitted electrons the transition rate w has to be integrated over the phase space of the neutrino and the electron. The total decay rate Γ is given by

$$\Gamma = \int \frac{V d^3p}{(2\pi\hbar)^3} \int \frac{V d^3q}{(2\pi\hbar)^3} \, w \, . \tag{5.1}$$

It is convenient to use polar coordinates for the momenta $\vec{p}$ and $\vec{q}$ and to apply the dispersion relations (2.16) and (2.23) for the electron and the neutrino. This yields

$$d^3p = m\sqrt{2mE}\,\sin\vartheta\, dE\, d\vartheta\, d\varphi ,$$

$$d^3q = \frac{1}{c^3} E_\nu^2 \sin\vartheta_\nu\, dE_\nu\, d\vartheta_\nu\, d\varphi_\nu .$$

For the transition rate w in Eq. (5.1) we use the semiclassical limit (3.10) which was shown to be an excellent approximation. Since w (3.10) depends only on E_ν and via z and n_0 on E and θ the integrals over θ_ν, ϕ_ν and ϕ are trivial in Eq. (5.1). After substituting E_ν by

$$x = n_0 + \frac{E_\nu}{\hbar\omega}$$

Γ takes the form

$$\Gamma = \frac{\sqrt{2m^3}}{4\pi^4\hbar^7 c^3}(\hbar\omega)^2 |g V_{fi}|^2 \int dE\, d\vartheta \sqrt{E}\,\sin\vartheta \times$$
$$\times \int_{n_0}^{\infty} dx\,(x-n_0)^2 \frac{\theta(z^2-n^2)}{\sqrt{z^2-n^2}} .$$

We have two restrictions for the integration variable x which have to be satisfied simultaneously: (i) $n_0 \leqslant x$, corresponding to the condition of positive neutrino energy $E_\nu > 0$, and (ii) $-z \leqslant x \leqslant z$ originating from the θ-function. Therefore we have to distinguish between three cases: (i) if $n_0 > z$ the total decay rate Γ vanishes; (ii) if $-z \leqslant n_0 \leqslant z$ the x-integration extends from n_0 to z; (iii) if $n_0 < -z$ the x-integration extends from -z to +z. This leads to the elementary integrals

$$\int_{n_0}^{z} dx \frac{(x-n_0)^2}{\sqrt{z^2-x^2}} = -\frac{3}{2} n_0 \sqrt{z^2-n_0^2} + \left(n_0^2 + \frac{z^2}{2}\right)\left(\frac{\pi}{2} - \arcsin\frac{n_0}{z}\right),$$

$$\int_{-z}^{z} dx \frac{(x-n_0)^2}{\sqrt{z^2-x^2}} = \pi\left(n_0^2 + \frac{z^2}{2}\right).$$

The angle and energy distribution of the electrons then reads

$$\frac{d^2\Gamma}{dE\,d\vartheta} = \frac{\sqrt{2m^3}}{4\pi^4\hbar^7c^3}\,|g\,V_{fi}|^2(\hbar\omega)^2\sqrt{E}\,\sin\vartheta\;\theta(z-n_0)\;\times$$

$$\times\begin{cases} -\frac{3}{2}n_0\sqrt{z^2-n_0^2}+\left(n_0^2+\frac{z^2}{2}\right)\left(\frac{\pi}{2}-\arcsin\frac{n_0}{z}\right) & \text{if } -z\le n_0\le z, \\ \pi\left(n_0^2+\frac{z^2}{2}\right) & \text{if } n_0<-z. \end{cases} \tag{5.2}$$

Since n_0 and z depend on E and θ, the condition $z \geqslant n_0$, i.e.

$$E+\frac{\nu^2}{2}mc^2-Q \le c\nu\sqrt{2mE}\,\sin\vartheta$$

yields the limits of allowed emission angles θ and of possible electron energies E. For $E \leqslant Q - \nu^2mc^2/2$ this condition is always satisfied. But for higher energies electrons are only emitted into a certain angular range. The limits for the emission angle are for fixed energy E

$$\sin\vartheta \ge \begin{cases} 0 & \text{if } E\le Q-\frac{\nu^2}{2}mc^2, \\ \dfrac{E+\frac{\nu^2}{2}mc^2-Q}{c\nu\sqrt{2mE}} & \text{if } E> Q-\frac{\nu^2}{2}mc^2. \end{cases} \tag{5.3}$$

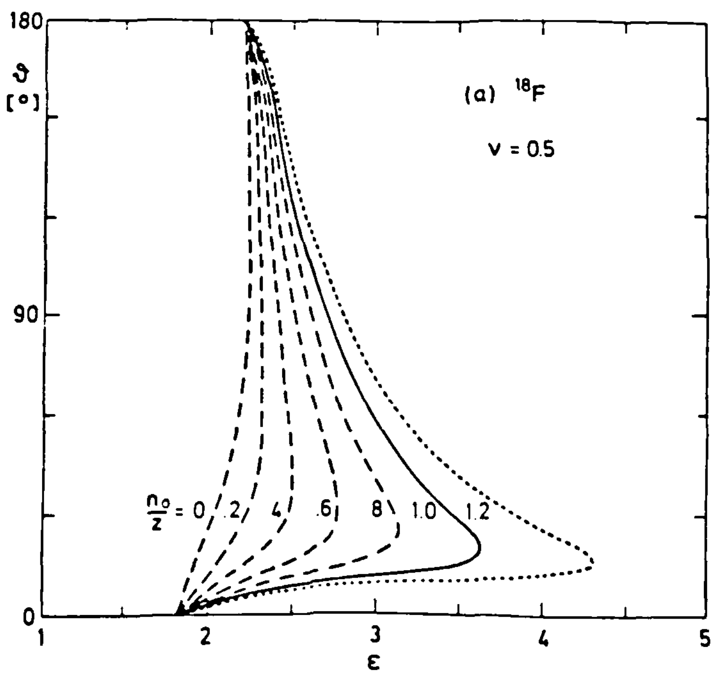
The limits for the electron energy can be obtained from the condition $\sin\theta \leqslant 1$:

$$E_{min} = \begin{cases} 0 & \text{if } Q\ge\frac{\nu^2}{2}mc^2 \\ \left(\sqrt{\frac{\nu^2}{2}mc^2}-\sqrt{Q}\right)^2 & \text{if } Q<\frac{\nu^2}{2}mc^2, \end{cases} \tag{5.4}$$

$$E_{max} = \left(\sqrt{\frac{\nu^2}{2}mc^2}+\sqrt{Q}\right)^2.$$

Figure 1 shows for the relativistic theory the possible (E,θ) values of the emitted electrons in a laser field with an intensity parameter $\nu = 0.5$ for two examples. ^{18}F decays via β^+ decay to ^{18}O with a moderate energy release Q = 634 keV, ^3H decays via β^- decay to ^3He with the very small energy release Q = 18.6 keV. (Note that we defined Q as the mass difference between nuclei, not between neutral atoms). The solid line in Fig. 1 shows the boundary of the integration area: on the left or inside the solid line, respectively, we have the allowed (E,θ) values. On the right or outside this curve $n_0 = z$, respectively, we have the nonclassical regime with extremely few events which are neglected in the present classical theory. Whereas Eq. (5.3) yields a symmetric electron distribution around $\theta = 90°$ with respect to the laser beam axis, the relativistic calculation yields an asymmetric distribution. Electrons at the high energy end of the spectrum are mainly emitted in forward direction. The dotted lines indicate a value of $n_0/z > 1$ up to which the Bessel functions are already so extremely decreased that this area cannot play a role in the nonclassical regime anymore. Nonclassical corrections to Eq. (5.2) can only be due to (E,θ) points extremely close to the solid line. Furthermore, the dashed lines in Fig. 1 show contours of constant ratios $n_0/z < 1$. The smaller this ratio is at some point (E,θ), the higher is the number of events that contribute to $d^2\Gamma/dEd\theta$ at this point.

For the example ^3H a laser field with $\nu = 0.5$ is already so strong that the effective mass exceeds the Q value. For all



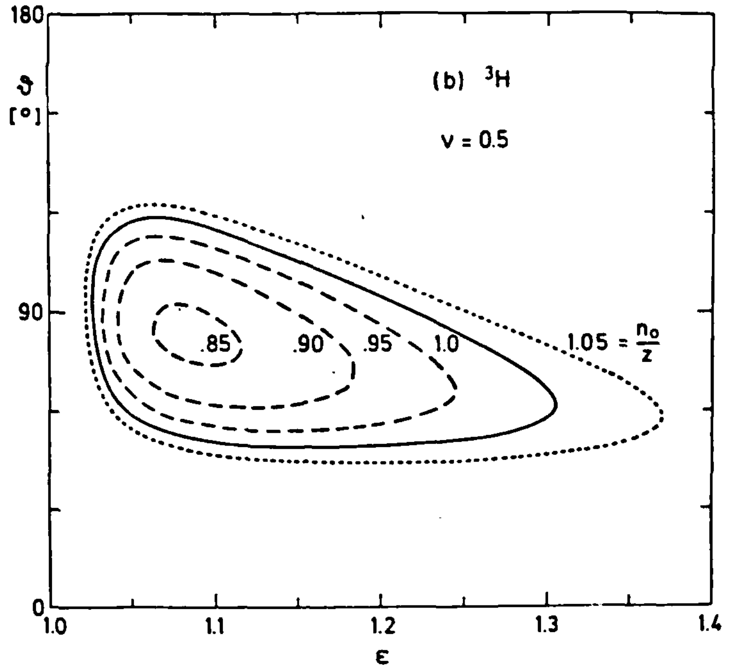


Fig. 1. Area of allowed emission angle and energy (solid curve) and contours of constant ratio n_0/z for a field intensity $\nu = 0.5$ and for the nuclei (a) ^{18}F and b) ^3H. The relativistic electron energy is scaled in units of mc^2: $\varepsilon = (m^2c^4 + p^2c^2)^{1/2}/mc^2$. These curves come from the complete relativistic theory.

energies n_0 is then positive. This is in contrast to the field-free situation where n_0 is always negative. The given combination of Q and ν allows only configurations with $n_0/z > 0.83$. The value n_0 denotes the minimum number of photons which the electron has to absorb from the field. In this example the electron must absorb energy from the field since the energy release of the nuclear decay alone is not sufficient to account for the effective mass of the electron in the field. Since n_0 for all (E, θ) values is rather close to z, only relatively few multiphoton terms contribute to $d^2\Gamma/dEd\theta$ at one particular (E, θ) point.

All these effects can also be seen in the spectrum of the electrons. We obtain the energy distribution of the electrons by integrating Eq. (5.2) numerically over θ between the limits (5.3). Figure 2 shows the results of the corresponding relativistic calculation, again for the two examples ^{18}F and ^3H. A similar plot can be found in Ref. 27. With increasing field intensity the electron distribution extends to much higher energies. On the other hand, the maximum of the spectrum

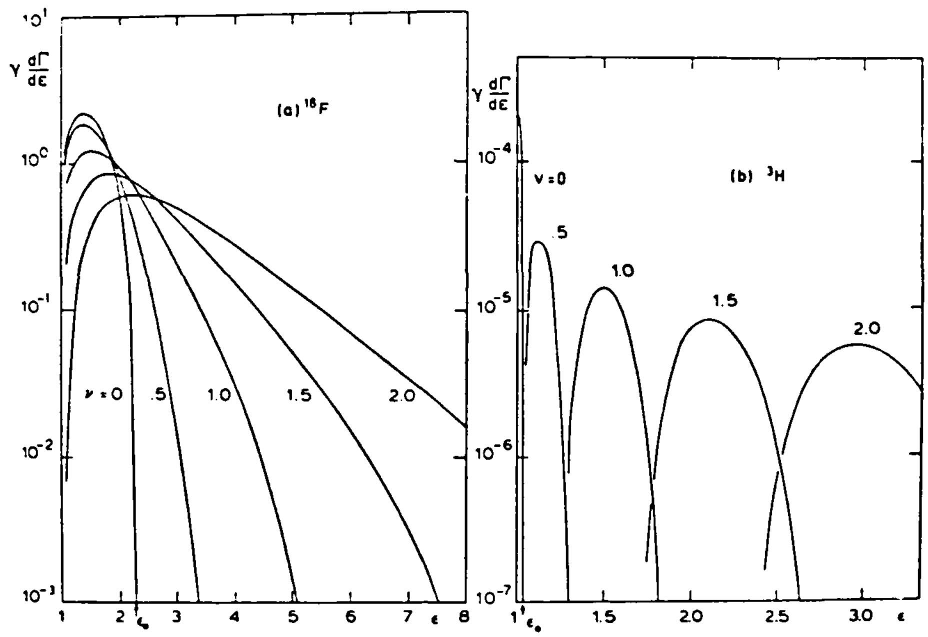


Fig. 2. Energy distribution of the electrons for different field intensities for the nuclei a) ^{18}F and b) ^{3}H. The relativistic electron energy is scaled in units of mc^2 as in Fig. 1, ϵ_0 is related to the Q value of the decay by $\epsilon_0 = 1 + Q/mc^2$, and γ denotes a constant factor of dimension (energy x time). These curves are calculated from the complete relativistic theory.

decreases with increasing ν. The area under the curve, which represents the total number of emitted electrons and therefore the lifetime of the nucleus, seems to be more or less constant for the different field intensities. We shall investigate this question in the next Section.

In the ^3H spectrum in Fig. 2b we see again the effect of the strong effective mass $\nu^2 mc^2/2$ exceeding the Q-value of the reaction. For sufficiently high ν-values the spectrum does not begin at E = 0 anymore, but at some finite energy. This reflects again the fact that the electrons from the low energy end of the laser-free spectrum must absorb a considerable amount of photons from the field in order to raise their effective mass, since the available Q value is not sufficient.

NUCLEAR LIFETIME

One possibility to obtain the total decay rate Γ and the lifetime of the nucleus $\tau = 1/\Gamma$ is the numerical integration of Eq. (5.2) between the limits (5.3) for the angle and (5.4) for the energy. However, it is also possible to calculate Γ analytically from Eq. (3.10) by choosing another set of variables for the electron momentum and another order of integration. The dependence of z on $(p_1^2 + p_2^2)^{1/2}$ suggests, for example, the following set of variables

$$s = p_1^2 + p_2^2 \,, \quad \varphi = \arccos \frac{p_1}{\sqrt{p_1^2 + p_2^2}} \,, \quad p_3$$

in terms of which

$$d^3p = \frac{1}{2}\, ds\, d\varphi\, dp_3 \,.$$

For the neutrino momentum $\vec{q}$ we use again polar coordinates. With this choice for $d^3\vec{p}$ and d^3q and with the semiclassical transition rate (3.10) the total decay rate Γ (5.1) reads after carrying out the three trivial angular integrations

$$\Gamma = \frac{|gV_{fi}|^2}{8\pi^4 \hbar^8 c^3 \omega} \int_0^{\infty} dE_\nu\, E_\nu^2 \int_{-\infty}^{\infty} dp_3 \int_0^{\infty} ds \tag{6.1}$$
$$\times\ \theta\left(z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2\right)\left(z^2 - \left(n_0 + \frac{E_\nu}{\hbar\omega}\right)^2\right)^{-1/2} .$$

After expressing z and n_0 in terms of s and p_3 the θ-function yields the following restrictions for the integration limits if the integrations are carried out in the indicated order (first ds, then dp_3 and finally dE_ν):

$$s_\pm = m^2\nu^2c^2 + 2m(Q-E_\nu) - p_3^2 \pm 2m\nu c\sqrt{2m(Q-E_\nu) - p_3^2} \geq 0 \,,$$

$$(p_3)_\pm = \pm\sqrt{2m(Q-E_\nu)} \,,$$

$$(E_\nu)_+ = Q \,.$$

The total decay rate Γ can then be rewritten in the form

$$\Gamma = \frac{m\,|g V_{fi}|^2}{4\pi^4\hbar^7 c^3} \int_0^Q dE_\nu\, E_\nu^2 \int_{-\sqrt{2m(Q-E_\nu)}}^{\sqrt{2m(Q-E_\nu)}} dp_3 \times \int_{s_-}^{s_+} \frac{ds}{\sqrt{(s_+-s)(s-s_-)}} \quad (6.2)$$

The value of the integral

$$\int_{s_-}^{s_+} \frac{ds}{\sqrt{(s_+-s)(s-s_-)}} = \pi$$

is independent of the intensity parameter ν, and so are the limits of the remaining integrations over p_3 and E_ν. Hence, after integration over ds the dependence on the external field drops out of the expression (6.2) for the total decay rate. This means that in the quasiclassical limit the total decay rate Γ is not affected by the external field, nor are the partial decay rates $d\Gamma/dp_3$, $d\Gamma/d\phi$ and $d^3\Gamma/d^3q$. Field effects only show up in $d\Gamma/ds$ and in the related partial rates $d\Gamma/dE$, $d\Gamma/d\theta$, $d\Gamma/dp_1$, $d\Gamma/dp_2$, etc.

The last two integrations in Eq. (6.2) over dp_3 and dE_ν are easily carried out. We then obtain for Γ the nonrelativistic limit of the total decay rate of allowed nuclear beta decay in the absence of an external field:

$$\Gamma_{free} = \frac{\sqrt{2m^3}}{2\pi^3\hbar^7 c^3} |g V_{fi}|^2 \int_0^Q dE \sqrt{E}\,(Q-E)^2$$
$$= \frac{8\sqrt{2}}{105\,\pi^3 c^6 \hbar^7} (mc^2)^{3/2} Q^{7/2} |g V_{fi}|^2 . \quad (6.3)$$

Again, there is a simpler way to realize the field independence of Γ in the classical limit: according to Eqs. (4.5-7)

$$\int w\, d^3p = \frac{2\pi}{\hbar V^2} |g V_{fi}|^2 \frac{\omega}{2\pi} \times \int_{t_0}^{t_0+2\pi/\omega} dt \int d^3p\, \delta\left(Q - E_\nu - \frac{1}{2m}(\vec{p} - e\vec{A}(t))^2\right), \quad (6.4)$$

where the integrations over t and $\vec{p}$ have been commuted. If we now transform the integration variable $\vec{p}$ to $\vec{p} - e\vec{A}(t)$, the field dependence is entirely eliminated. Obviously, in order to achieve this, it suffices already to integrate over the two components of $\vec{p}$ which lie in the plane of $\vec{A}(t)$.

The relativistic theory also leads to the result that the nuclear lifetime is unaffected by the external field. The physical importance of the effective mass term $\nu^2 mc^2/2$ for this result should be stressed again. We recall that the effective mass is due to the $\vec{A}^2$-term of the interaction Hamiltonian in the R-gauge. If the field is strong enough so that the effective mass exceeds the Q value of the decay emitted electrons must absorb the energy difference from the field. We thus have two competing processes: since the electrons absorb energy they have a larger phase space and the nucleus should decay faster. On the other hand, due to the effective mass only those electrons are emitted which absorb between n_0 and approximately z photons. The distribution of the photon absorption with respect to the photon number n is given in the classical limit by the approximation (3.9) for the Bessel functions or by the classical distribution (4.6). With increasing ν, i.e. with increasing phase space, the minimum of n_0/z in the entire (E,θ) plane approaches unity so that the portion $(z - n_0)z$ of photon absorptions that still leads to decay becomes very small. In the limit of classical electron field interaction these two effects, increasing the phase space and overcoming the effective mass, cancel out each other exactly.

In a different context, namely the hyperfine splitting of atomic levels in a strong radio frequency field, the just mentioned cancellation has been noticed previously: by merely replacing the electron mass m by the effective mass $(m^2+e^2\vec{A}^2/c^2)^{\frac{1}{2}}$ in the Hamiltonian, an effect was predicted which was not corroborated by experiment [33]. It was later shown [34], that in the long wave length limit this effect is cancelled by the $\vec{p}\vec{A}$-term which was initially neglected. This is another example for the interplay between the $\vec{p}\vec{A}$- and the $\vec{A}^2$- term. The argument following Eq. (6.3) shows clearly that this cancellation is just a consequence of the minimal coupling interaction.

However, one should keep in mind that the field-independence of the nuclear lifetime holds only in the classical limit $\hbar \to 0$ for the electron-field interaction. This limit is well justified for all beta decaying nuclei in the presence of laser fields with intensity ν in the order of unity and with optical or longer wavelengths. However, the quasiclassical limit breaks down for fields with shorter wavelengths (x-ray regime) and for (fictitious) nuclei with tiny Q values $Q \ll mc^2$. In these cases the parameter z which corresponds to the maximum number of absorbed photons cannot be considered a large number anymore. The classical approximation, however, holds only for multiphoton

transfers of very high order. The classical limit furthermore breaks down for superintense fields when $\nu \gg 1$. Then the effective mass term becomes so large that the minimum value of the parameter n_0, which corresponds to the minimum number of absorbed photons, comes very close to z, i.e. there are no (E,θ) configurations with $n_0 \ll z$ anymore. In the Bessel function procedure this implies that (E,θ) configurations beyond the $n_0 = z$ border become important. For the stationary phase method the case $\nu \gg 1$ means that higher order terms in the expansion around the stationary points have to be taken into account. These three ways to get away from the validity of the semiclassical limit will be demonstrated again in the next Section.

THE NONCLASSICAL REGIME

In order to calculate the quantum effects of the electron-field interaction we have to attempt to calculate the total decay rate analytically as far as possible without any approximations. We therefore start from the exact expression (3.7) for the transition rate per time in terms of Bessel functions and choose again the variables (s,ϕ,p_3) for the electron momentum as in the last section. The sum over the Bessel functions in Eq. (3.7) cannot be carried out anymore. We choose the order of integrations so that we avoid an integration over the Bessel function. The three integrations over the angular variables are trivial, as well as the integrations over the neutrino energy E_ν and over p_3. We thus obtain an exact expression for the nuclear lifetime in a laser field

$$\Gamma = \frac{1}{30\pi^3 c^3 \hbar^7} \frac{|g V_{fi}|^2}{m^2} \sum_{n=-\infty}^{\infty} \int ds\, J_n^2\left(\frac{\nu c}{\hbar\omega}\sqrt{s}\right) \times \tag{7.1}$$

$$\times \left(2m\left(Q + n\hbar\omega - \frac{\nu^2}{2}mc^2\right) - s\right)^{5/2} \theta\left(2m\left(Q + n\hbar\omega - \frac{\nu^2}{2}mc^2\right) - s\right).$$

The summation over n and the integration over s can be carried out analytically. For this purpose the sum of the squared Bessel functions has to be transformed in a proper way, and the calculation of the total decay rate Γ is somewhat lengthy [28].

However, there also exists a completely different approach for the exact calculation of the total decay rate Γ in the presence of an external field. It is possible to express the field influence on Γ by an operator acting on the decay rate Γ_{free} (6.3) of the beta decay in the absence of the field [29]. The exact expression for the nonrelativistic rate Γ of allowed beta decay reads

$$\Gamma = \exp\left\{-\frac{i}{\hbar} M\left(-i\hbar \frac{\partial}{\partial Q}\right)\right\} \Gamma_{free} . \tag{7.2}$$

For circularly polarized fields the operator M has the form

$$M(x) = \frac{\nu^2}{2} mc^2 x \left\{ 1 - \left(\frac{\sin\frac{\omega x}{2}}{\omega x/2}\right)^2 \right\} . \tag{7.3}$$

As usual, these operators are defined by their power series expansion

$$\exp\left\{-\frac{i}{\hbar} M\left(-i\hbar\frac{\partial}{\partial Q}\right)\right\} = \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \left\{-\frac{i}{\hbar} M\left(-i\hbar\frac{\partial}{\partial Q}\right)\right\}^{\ell}$$

and

$$-\frac{i}{\hbar} M\left(-i\hbar\frac{\partial}{\partial Q}\right) = \frac{\nu^2}{2} \sum_{k=1}^{\infty} \frac{c_k}{4^k} \left(mc^2\frac{\partial}{\partial Q}\right)\left(\hbar\omega\frac{\partial}{\partial Q}\right)^{2k} .$$

The coefficients c_k are given by

$$c_k = \sum_{n=0}^{k} \frac{1}{(2n+1)!\,(2(k-n)+1)!} .$$

Noticing that x occurs in Eq. (7.3) in combination with ω as the product ωx, we can simplify the operator considerably. The k-th term of the operator M applied to the free decay rate Γ_{free} differs by the order $(\hbar\omega/Q)^2$ from the (k-1)-th term. In the limit $\hbar\omega \ll Q$ it is therefore sufficient to take only the first term k = 1 in M into account. Consistently also $\exp\{-iM/\hbar\}$ can then be cut off at $\ell = 1$. To first order of $(\hbar\omega/Q)$ the total decay rate is thus given by

$$\begin{aligned} \Gamma &\simeq \left\{ 1 + \frac{\nu^2}{24}\left(mc^2\frac{\partial}{\partial Q}\right)\left(\hbar\omega\frac{\partial}{\partial Q}\right)^2 \right\} \Gamma_{free} \\ &= \left\{ 1 + \frac{35}{64} \frac{mc^2}{Q}\left(\frac{\hbar\omega}{Q}\right)^2 \nu^2 \right\} \Gamma_{free} . \end{aligned} \tag{7.4}$$

The enhancement of the total decay rate due to the external field is then to first order of $(\hbar\omega/Q)$

$$R = \frac{\Gamma}{\Gamma_{free}} = 1 + \frac{35}{64} \frac{mc^2}{Q} \left(\frac{\hbar\omega}{Q}\right)^2 \nu^2 . \tag{7.5}$$

Expressing the intensity parameter ν by the critical field strength E_c (2.23) the enhancement can be written in the form

$$R = 1 + \eta(Q)\left(\frac{E_0}{E_c}\right)^2 , \quad \eta(Q) = \frac{35}{64}\left(\frac{mc^2}{Q}\right)^3 . \tag{7.6}$$

This result agrees with the nonrelativistic limit in Ref. 28. As mentioned at the end of the preceding Section we can obtain a considerable enhancement either for nuclei with very small Q values $Q \ll mc^2$, or for x-ray fields or for high field intensities $\nu > 1$.

Figure 3 shows the enhancement R (7.5) as a function of ν for a Nd laser with $\hbar\omega = 1.17$ eV and for several Q-values. For $Q \ll mc^2$ the enhancement already becomes very large for relatively moderate field strengths. It should, however, be mentioned that the chosen Q values are considerably smaller than the Q values of any existing beta decaying nuclei. For ^3H Eq. (7.5) requires already $\nu = 4\times10^3$ for a Nd laser in order to obtain R = 2, corresponding to a field strength E_0 of 1.2×10^{14} V/cm!

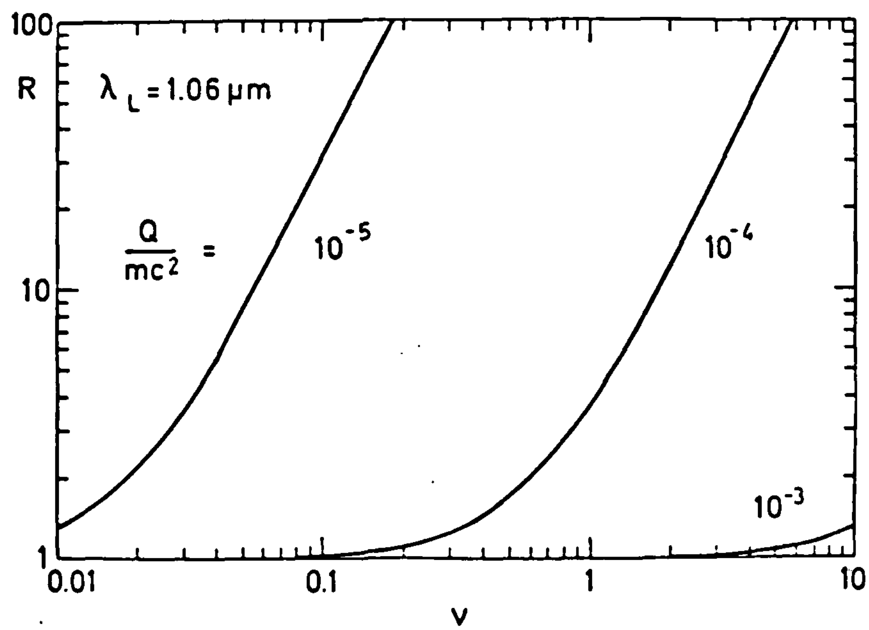


Fig. 3. Enhancement R of the total decay rate Γ due to an external field of wavelength $\lambda = 1.06\ \mu m$ as a function of the intensity parameter ν for different Q values.

It is interesting to note that in the relativistic theory in first order of $(\hbar\omega/mc^2)$ the enhancement R still has the form (7.6). Just the function η which depends only on the Q value of the decay has to be replaced by a relativistic expression [28]. This implies that the field dependence of the enhancement R is fairly well described by the nonrelativistic theory in Eqs. (7.5) or (7.6) for all field parameters. Hence the nonrelativistic theory yields also for $\nu \gg 1$ a good estimate of the total decay rate which justifies its use for Fig. 3. Furthermore, the nonrelativistic total decay rate also gives for the entire range of Q-values of realistic nuclei approximately the correct answer, i.e. it also holds for electron energies $E \gg mc^2$. We can see this from Fig. 4 which compares the Q-dependent factor of Eq. (7.6) in the relativistic and in the nonrelativistic theory. For $Q/mc^2 < 0.1$ both curves of η are identical and up to $Q/mc^2 \lesssim 100$ they never differ by more than a factor of 4. This proof that the nonrelativistic theory yields reliable enhancements even for $\nu \gg 1$ and $Q \gg mc^2$ is the physical justification for its use throughout these lecture notes. However, there are, of course, measurable relativistic effects in the angular and in the energy distribution of the electrons and polarization effects are not at all accounted for by the nonrelativistic treatment.

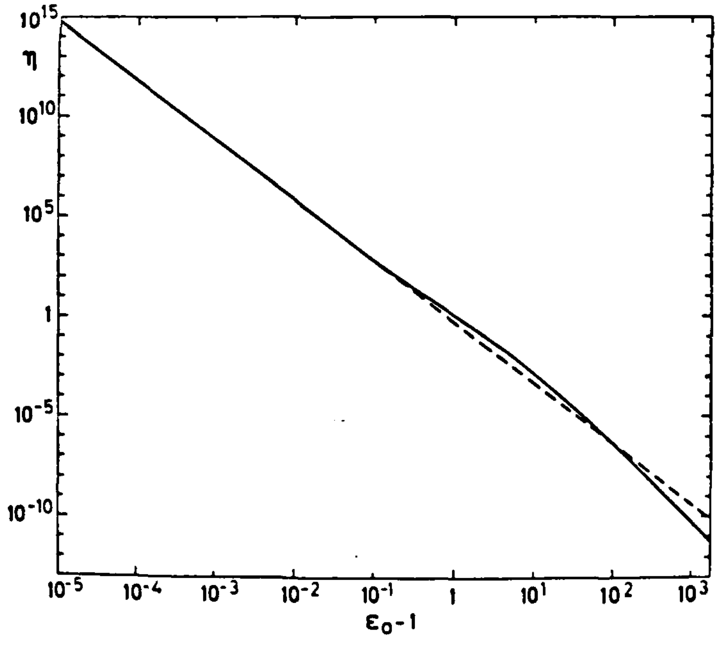


Fig. 4. Q-value factor η (7.6) in the Dirac theory (solid curve) and in the Schrödinger theory (dashed line) as a function of $Q/mc^2 = \varepsilon_0 - 1$.

SUMMARY

We have shown that the modification of the beta-decay lifetime due to an external electromagnetic field is a pure quantum effect. For presently feasible laser fields the interaction between the emitted electron and the field is well described by the classical limit $\hbar \to 0$, so that the nuclear lifetime is not affected. In order to obtain appreciable quantum effects in nuclear beta decays either (i) the photon energy has to be of the order of the Q value of the decay [30] or (ii) the field strength must be comparable with the critical field strength. We have furthermore seen in Fig. 4 that the modification of the lifetime is to a fair approximation a nonrelativistic quantum effect. Relativistic quantum corrections are small for all realistic nuclei.

Crucial for the result that the classical electron-field interaction does not give rise to a change in the nuclear lifetime, is the interplay between the $\vec{p}\vec{A}$-term of the interaction which is responsible for the energy exchange between the electron and the field, and the $\vec{A}^2$-term which contributes to the effective mass. For intense field problems the $\vec{A}^2$-term is very important and cannot be neglected as it is often done in quantum optics.

The result found here contradicts directly Refs. 10 and 14, which predicted a considerable enhancement of free neutron and nuclear beta decay rates with lasers available at present and casts doubt on Refs. 13 and 15 in which similar enhancement for γ decay and forbidden beta decay are obtained. We can state that the numerical results in all of these four publications are (due to completely different reasons) incorrect. In Refs. 10 and 14 the "enhancement" of the decay rate is due to a careless handling of sums over Bessel functions [25] as was discussed in these lecture notes. In Ref. 13 the underlying physical situation is not correctly modeled [31] and the calculations in Ref. 15 are misguided by a wrong interpretation of the non-interacting nuclear wave function (2.13) and include an algebraical error, after whose correction the proposed enhancement of forbidden decay rates disappears [32]. Obviously, Nature does not want her more elementary constituents to be tampered with by inadequate means.

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kinetical momentum $\vec{p} - e\vec{A}$. In order to double check, we notice from Eq. (3) that the form factor satisfies this requirement. The function $E'(u)$ can be rewritten as

$$E'(u) = E_f - E_i + q_0 + p_0 - \frac{e}{2(p_0 - p_z)} (2\vec{p}\vec{A}(u) - e\vec{A}^2(u))$$

$$= E_f - E_i + q_0 + \frac{1}{2} (p_0 - p_z) + \frac{m^2 + (\vec{p}_T - e\vec{A}(u))^2}{2(p_0 - p_z)} \tag{7}$$

where we used the mass shell condition $(p_0 - p_z)(p_0 + p_z) = m^2 + \vec{p}_T^2$. Hence $|M|^2$ depends only on the kinetical momentum, as it should.*

We note in passing that the same holds true if the electron satisfies the Dirac equation. The Volkov solution is then augmented by the spinor

$$(1 + e\frac{\not{n}\not{A}}{2pn})\, u_p \ ,$$

which, since u_p satisfies $(\not{p} - m)u_p = 0$, can be rewritten as

$$(1 - \frac{\not{n}}{2pn} (\not{p} - e\not{A} - m))u_p \ .$$

* Actually, we have defined p as the momentum of the electron outside of the field rather than the canonical momentum. Let us denote the latter for the moment by p_c. We conclude from the fact that the external field depends only on u, that the components $\vec{p}_{cT}$ and $p_{c0} - p_{cz}$ are conserved. In particular, they do not change when the field, as a function of u, is turned on or off. Hence, inasmuch as these components are concerned and only these do occur, we can identify the asymptotic with the canonical momentum.

Now, in order to get the total decay rate, the differential rate $|M|^2$ has to be integrated over the momenta of the decay products. If we change the integration variable from $\vec{p}$ to $\vec{p} - e\vec{A}(u)$ and commute the integration over $\vec{p}$ with that one over u, the dependence of the external field is entirely eliminated.

Let us compare the preceding argument with our earlier note regarding the decay of a neutral free particle. The only difference lies in the fact that in the latter case the integral over space could be explicitly carried out yielding a delta-function as the expression of momentum conservation. In the present case, owing to the presence of the nuclear wave functions, this was no longer possible and we were left with the form factor (4). The decisive point of the argument, the dependence of the instantaneous decay rate on the kinetical momenta, is exactly the same in both cases.

The above procedure depends crucially on the fact that we were able to write the total rate, within the quasiclassical approximation, as an integral over an instantaneous rate, as represented in Eq. (6). We expect that this approximation will break down as soon as the external field is so strong that the change in the kinetical momentum during the decay time t is comparable with the electron rest mass, viz.

$$\Delta(|\vec{p} - e\vec{A}(t)|) \sim e|\vec{E}(t)|\Delta t \sim m \ . \tag{8}$$

From the uncertainty relation we estimate $\Delta t \sim \hbar/E_0$ with $E_0 = E_i - E_f$ the nuclear energy release, so that we expect our previous considerations to apply as long as

$$\frac{m|\vec{E}|}{E_0 E_{crit}} << 1 . \qquad (9)$$

In fact, calculated enhancements of allowed [9] as well as forbidden [6] nuclear beta-decay are proportional to $(m/E_0)^3(|\vec{E}|/E_{crit})^2$ in rough agreement with the hand-waving estimate (9). To give an explicit example we obtain for the first forbidden decay of ^{90}Sr in the presence of a radiofrequency field with $|\vec{E}|/E_{crit} = 6 \times 10^{-13}$ and a wave length of $\lambda = 100m$ an enhancement of 2×10^{-24}, i.e. zero for all practical purposes in agreement with our above argument. In contrast, in Ref. 2 a value of the order of unity is given.

To summarize, we have shown via a very simple, general and fully relativistic argument that nuclear beta-decay, forbidden or not, cannot be enhanced by experimentally feasible radiation fields. The situation is particularly disillusioning for the radio frequency fields with $|\vec{E}|/E_{crit} \sim 10^{-12}$, which were considered in Ref. 2. It is different for high-intensity x-ray lasers which are nearly resonant with nuclear energy level differences [10], since in this case the interaction between the nucleus and the field is of paramount importance.

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